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A brief note on the self-balanced singular elastic field caused by an edge dislocation

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ABSTRACT

Four physical quantities describing the self-balanced singular elastic field caused by an edge dislocation before or after loading are obtained by using SW-integral. These quantities are directly related to mechanisms of dislocation plasticity and are compared with that of a concentrated force. An edge dislocation and a concentrated force interacting with the first stress invariant and the maximum shear stress at infinity, respectively, are also presented.
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1. Introduction

Traditional conservation integrals, such as *J*-integral, *L*-integral and *M*-integral, play an important role in finding out the physical quantities of material points with singularities and the interaction among singularities in elastic field [1–5]. For example, apart from the crack extension force, Cherepanov [6] gives the known Peach–Koehler formula by calculating *J*-integral around an edge dislocation as follows

$$J_i = e_{ij3}\sigma_{jk}^{\infty}b_k, (i,j,k = 1,2), \tag{1}$$

where $(b_1, b_2) = b(\cos\gamma, \sin\gamma)$ is the Burgers vector of an edge dislocation, σ_{jk}^{∞} are the stresses at infinity and e_{ij3} is the permutation symbol. The formula (1) means a configuration force acting on the dislocation. However, when letting $\sigma_{jk}^{\infty} = 0$ at infinity, we know that $J_i = 0$ in formula (1). That is, it is difficult for us to know whether there exists a self-balanced singular elastic field caused by an edge dislocation by using *J*-integral without loading.

It is well known that mechanisms of dislocation plasticity are directly related to the self-balanced singular elastic field caused by edge dislocations [7]. As mentioned in [8], dislocation motion on the primary slip system is severely hindered by the presence of dislocations on the secondary slip systems, which is the reason for the high stage-II hardening rate in materials caused by the possibility of secondary slips. Therefore, it is significant to recognize the characteristics of dislocations before or after loading.

Recently, it is found that for any analytic function $\phi(z)$, there exists SW-integral in the sense of Noether's theorem [9–12]

$$\oint_{\Gamma} \zeta(z)[\phi'(z)]^2 dz = 0. \tag{2}$$

Here, $\zeta(z)$ represents any conformal transformations, so that there are countless conserved quantities and path-independent integrals. Clearly, by adjusting the conformal transformation $\zeta(z)$, a finite physical quantity can always be obtained when calculating SW-integral (2) around a material point with any order singularity.

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In this note, an edge dislocation with or without the loading at infinity is considered by using SW-integral (2) and some new physical quantities are obtained. For comparison, a concentrated force acting at the origin is also calculated. The purpose is to show some light on edge dislocations with the measurement before or after loading.

2. An edge dislocation

Under the consideration of a plane stress or plane strain problem, suppose that an elastic body contains an edge dislocation. The complex Kolosov–Muskhelishvili potentials $\phi(z)$ and $\psi(z)$ are given by [6,13]

$$\phi(z) = \frac{G(b_1 + ib_2)}{\pi i(\kappa + 1)} \ln z + A_1 z, \tag{3a}$$

$$\psi(z) = -\frac{G(b_1 - ib_2)}{\pi i(\kappa + 1)} \ln z + (B_1 + iB_2)z, \tag{3b}$$

$$A_1 = (\sigma_{11}^{\infty} + \sigma_{22}^{\infty})/4, B_1 = (\sigma_{22}^{\infty} - \sigma_{11}^{\infty})/2, B_2 = \sigma_{12}^{\infty}, \tag{3c}$$

where G is the shear modulus, $(b_1 + ib_2) = b(\cos\gamma + i\sin\gamma)$ is a linear dislocation, $\kappa = 3 - 4v$ for plane strain, $\kappa = (3 - v)/(1 + v)$ for plane stress and v the Poisson's ratio. Please note that the rigid body translation and rotation have been neglected in (3a) and (3b), which do not affect elastic deformation.

Applying the SW-integral (2) to the complex potentials (3a) and (3b) in an area surrounded by the dotted lines shown in Fig 1(a), one has

$$SW^{(\phi D)} = \oint_{\Gamma_\infty} \zeta(z)\left\{\left[\frac{G(b_1 + ib_2)}{\pi i(\kappa + 1)z}\right]^2 + 2A_1\frac{G(b_1 + ib_2)}{\pi i(\kappa + 1)z} + A_1^2\right\}dz, \tag{4a}$$

$$SW^{(\psi D)} = \oint_{\Gamma_\infty} \zeta(z)\left\{\left[\frac{G(b_1 - ib_2)}{\pi i(\kappa + 1)z}\right]^2 - 2(B_1 + iB_2)\frac{G(b_1 - ib_2)}{\pi i(\kappa + 1)z} + (B_1 + iB_2)^2\right\}dz, \tag{4b}$$

where the integral paths Γ_0 and Γ_∞ are in an anticlockwise direction.

In order to obtain some physical quantities, the following basic integral will be used

$$\oint z^n dz = \begin{cases} 2\pi i, & n = -1 \\ 0, & n \neq -1 \end{cases} \tag{5}$$

By letting $\zeta(z) = z$, it is derived from (4a) and (4b) that

$$SW_z^{(\phi D)} = \frac{2(Gb)^2}{\pi(\kappa + 1)^2}(\sin 2\gamma - i\cos 2\gamma), \tag{6a}$$

$$SW_z^{(\psi D)} = -\frac{2(Gb)^2}{\pi(\kappa + 1)^2}[\sin 2\gamma + i\cos 2\gamma]. \tag{6b}$$

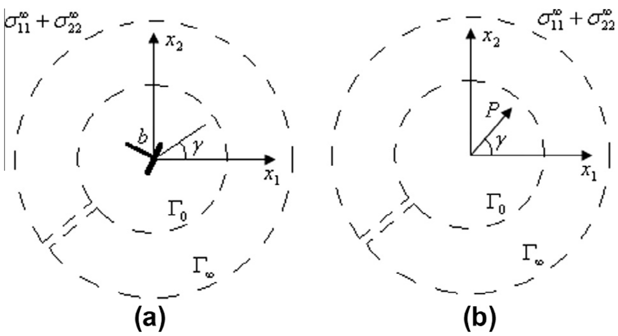

Fig. 1. (a) An edge dislocation b at the origin; (b) a concentrated force P acting at the origin.

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