Applied Mathematics and Computation 224 (2013) 46–53

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Modelling and analysis of repairable systems with preventive maintenance

Houbao Xu [a,*], Weiwei Hu [b]

[a] Department of Mathematics, Beijing Institute of Technology, Beijing 100081, PR China
[b] Department of Mathematics, University of Southern California, Los Angeles, CA 90089, USA

ARTICLE INFO

Keywords:
Preventive maintenance
Time delay equation
Difference scheme

ABSTRACT

The time-dependent solution of a kind of repairable system with preventive maintenance is investigated in the paper. With total probability formula, we show that the behavior of the system can be described as a group of ordinary differential equations coupled with partial differential equations, which can be formulated as a time delay equation in an appropriate Banach space. Based on the time-delay equation, this paper presents a difference scheme as an approximating method to solve the time-dependent solution which is necessary for analyzing the instantaneous availability of the repairable system. Some numerical examples are shown to illustrate the effectiveness of this approach.

1. Introduction

Traditional repairable systems assume a whole range of performance levels, varying from perfect functioning to complete failure, and assuming the repair is perfect. However, many manufacturing systems suffer increasing wear with usage as age as deteriorating process, that is, perfect functioning and repair is not always satisfied. Therefore, maintenance management as an important policy in repairable system is widely used to keep system in good condition to decrease failures and increase system availability.

Both preventive maintenance (PM) and corrective maintenance (CM) are important methods for deteriorating system to obtain the greatest availability. PM is defined as the activity undertaken regularly at pre-selected intervals while the system is satisfactorily operating, to reduce or eliminate the accumulated deterioration. While CM is the activity to bring the system to a state as good as new after it has experienced a failure.

Since a simple periodic replacement model with minimal repairable was presented by Barlow and Hunter [1], the analysis of optimal maintenance time of repairable system had drawn more and more attentions. Brown and Proschan [2] firstly suggested an imperfect repair model in which the system will be perfect repair with probability p or minimal repair with probability $1-p$. Sheu et al., [3] applied periodic preventive maintenance to different repair models and discussed the optimum PM time T^* to minimize the cost rate. Courtois and Delsarte [4] demonstrated the existence of an optimal interval between inspections, which maximizes the mean time between system failures. Also, the expression of the optimum was obtained to evaluate the influence of the parameters of system. Jia and Wu [5] investigated the replacement policy by using geometric process and renewal process to minimize the expected long-run cost per unit time. Later Wang and Zhang [6] studied a repair-replacement problem for a deterioration cold standby repairable system, and determined an optimal bivariate policy (T,N) by using geometric process to minimize the average cost rate of the system. Xu and Hu [7] studied the availability optimization of repairable system by the method of probability and strong continuous semigroup theory.

* Corresponding author.
E-mail address: xuhoubao@bit.edu.cn (H. Xu).

0096-3003/$ - see front matter © 2013 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.amc.2013.08.051

In [7], the optimal time to carry out preventive maintenance in terms of steady availability of the system was analyzed theoretically and numerical examples were also presented. Xu and Hu [8] investigated the convergence of a simple finite difference scheme which was used in simulating the instantaneous indexes of a system consisting of two machines separated.

All papers mentioned above aimed to derive optimal preventive maintenance schedule to maximize steady availability or other steady indexes. However, research on instantaneous availability is an important issue because it can provide us the measure required for the evaluation, such as, the availability of a system in certain time interval. On the other hand, instantaneous availability, in general, is difficult to obtain the exact expression (see [9]). So traditional analysis for availability focuses on steady-state availability, and assumes the steady-state availability as the system's availability. Such assumption, as we all know, does not always hold, especially for high reliable repairable systems, because it will cost a long time before the system reaches it steady-state. In the process, instantaneous availability even may have ripples. Fortunately, there are some literatures put emphasis on instantaneous availability. Sun and Han [10] studied the instantaneous availability of the system with time-varying failure rate. But the method shown in that paper is not a common way to solve instantaneous availability. Because they assumed that the failure rate should be a constant or decreasing or increasing function. Amiri and Tari [11] used a continuous time exponential Markov chain to describe the system and presented the instantaneous availability of a k-out-of-n system with the method of transition matrix. Such method is effective in dealing with Markov process, but it is powerless in the face of these systems which are described with semi-Markov Process. Cui and Xie [9] discussed the instantaneous availability function of two models with general distribution, but assuming that the system either in up-state or in down-state.

How to get the instantaneous availability of the repairable system is an interesting issue when the optimal preventive maintenance schedule is determined. Motivated by this, the paper devotes to finding a new approach to study the instantaneous availability of the repairable system with preventive and corrective maintenance. For self-contained, Section 2 uses mathematical model to formulate the repairable system; Section 3 presents an approximating model which make it possible for us to use finite difference scheme to investigate the time-dependent solution. Two numerical results of instantaneous availability of the repairable system are shown in Section 4; Section 5 concludes the paper.

2. System description and formulation

The system, discussed in this paper, transits through several stages as time progresses (see Fig. 1). The notations that will be used in this paper are listed as follows:

- State 0 is the best state. Both CM and major PM take the system back to this state;
- State 1 is the good state. System degradations start to occur but not a threat yet. Minimal PM will bring the system back to this state;
- State 2 is the doubtful state. PM is needed or a system crash may happen in this state. If a system crash happens, the system will enter state 3;
- State 3 is the down state. States 0, 1, 2 and 3 can be distinguished by condition-based monitoring;
- State 4 comes from state 2 with probability p. If the system enters state 4, minimal PM will be carried out immediately;
- State 5 comes from state 2 with probability $(1-p)$. If the system enters state 5, major PM will be carried out immediately;
- λ_i is the degradation rate in state $i, i = 0, 1$. Here, λ_i also can be regarded as parameters of exponential distributions;
- λ_3 is the Poisson failures rate from state 0 or from state 1 to state 3;
- $\lambda_2(x)$ is the failure rate of system crash in state 2. Here, x presents the sojourn time of system since it enters state 2 from state 1;
- μ_1 is the Minimal PM rate of the system from state 4 to state 1;
- μ_2 is the Major PM rate of the system from state 5 to state 0;

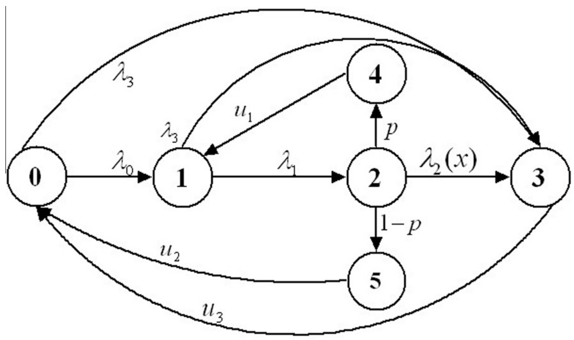

Fig. 1. states transmission of repairable system with PM and CM.