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Asymptotic behavior for a coupled Petrovsky and wave system with localized damping ☆

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ABSTRACT

In this paper we investigate asymptotic behavior for the solution of a coupled Petrovsky and wave system with locally distributed nonlinear interior damping. We establish a very general decay estimate for the energy by using an approach developed in Lasiecka and Tataru (1993) [23]. Using the energy decay formula, several explicit decay rate estimates can be obtained.

1. Introduction

In this paper we study energy decay rate estimate of the solution to the following initial and boundary value problem for the coupled Petrovsky and wave system with locally distributed nonlinear interior damping:

$$\begin{cases} u_{tt} + \triangle^2 u + a(x)v + \rho_1(x, u_t) = 0, & x \in \Omega, \quad t > 0, \\ v_{tt} - \triangle v + a(x)u + \rho_2(x, v_t) = 0, & x \in \Omega, \quad t > 0, \\ \frac{\partial u}{\partial \nu} = u = v = 0, & x \in \partial\Omega, \quad t > 0, \\ (u, v)(0) = (u_0(x), v_0(x)), \ (u_t, v_t)(0) = (u_1(x), v_1(x)), & x \in \Omega, \end{cases} \tag{1.1}$$

where Ω is an open bounded domain in $\mathbb{R}^n (n \geqslant 1)$ with smooth boundary $\partial\Omega = \Gamma$, ν is the outward unit normal vector on Γ; $a(\cdot) : \Omega \to \mathbb{R}$ and $\rho_i(\cdot,\cdot) : \Omega \times \mathbb{R} \to \mathbb{R}$ $(i = 1, 2)$ are given functions. We shall prove explicit energy decay rate estimates for the problem (1.1) under suitable assumptions on the functions $a(\cdot)$, $\rho_i(\cdot,\cdot)$, $i = 1, 2$ and the domain Ω.

The physical origin of problem (1.1) lies in the study of beam. Mathematical problems of type (1.1) have been investigated by several authors, for example, see the papers [14,15,20,21]. Our work is motivated by some works of Martinez [25], Alabau-Boussouira[1–3], Han and Wang [16] and Guessmia [14]. Martinez [25] studied the wave equation with localized damping:

$$\begin{cases} u_{tt} - \triangle u + \rho(x, u_t) = 0, & x \in \Omega, \quad t > 0, \\ u = 0, & x \in \Gamma, \quad t > 0, \\ u(0) = u_0, \ u_t(0) = u_1, & x \in \Omega, \end{cases} \tag{1.2}$$

where Ω is an open bounded domain in $\mathbb{R}^n (n \geqslant 1)$ with smooth boundary $\partial\Omega = \Gamma$, $\rho(\cdot,\cdot) : \Omega \times \mathbb{R} \to \mathbb{R}$ is a given function. With no polynomial growth condition on the damping terms, by using the piecewise multiplier method and some nonlinear

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integral inequalities, Martinez weakened the usual geometrical conditions on the location of the damping and proved precise explicit energy decay rates of the solution to (1.2).

In paper [1] Alabau-Boussouira considered the abstract version of (1.2):

$$\begin{cases} u_{tt} + Au + \rho(x, u_t) = 0, & t > 0, \\ u(0) = u_0, \ u_t(0) = u_1, \end{cases} \tag{1.3}$$

where A is a self-adjoint densely defined linear unbounded operators in H (H is a Hilbert space), $\rho(\cdot,\cdot) : \Omega \times \mathbb{R} \to \mathbb{R}$ is a given function. Under some suitable conditions on $\rho(\cdot,\cdot)$ and the domain Ω Alabau-Boussouira established a general semi-explicit formula for the energy decay rate of solutions to (1.3) provided that the energy satisfies a weighted integral inequality. Applications of this decay formula to single wave equations with nonlinear internal localized damping and boundary localized damping are also given in [1]. Recently, Alabau-Boussouira extended the energy decay result of [1] to a Petrovsky equation with locally distributed damping in [2] and to Timoshenko beams subject to a single nonlinear feedback in [3].

In Guesmia [14], Han and Wang [16] they considered (1.1) with globally distributed damping, i.e. the following problem:

$$\begin{cases} u_{tt} + \triangle^2 u + a(x)v + \rho_1(u_t) = 0, & x \in \Omega, \quad t > 0, \\ v_{tt} - \triangle v + a(x)u + \rho_2(v_t) = 0, & x \in \Omega, \quad t > 0, \\ \frac{\partial u}{\partial \nu} = u = v = 0, & x \in \partial\Omega, \quad t > 0, \\ (u, v)(0) = (u_0(x), v_0(x)), \ (u_t, v_t)(0) = (u_1(x), v_1(x)), & x \in \Omega. \end{cases} \tag{1.4}$$

With polynomial growth condition on $\rho_1(\cdot)$ and $\rho_1(\cdot)$ at the origin Guesmia established exponential and polynomial decay rate for the energy of solutions to (1.4). However, Han and Wang [16] eliminated this condition, with no growth conditions on the damping terms in zero they proved very general decay rate for the energy of (1.4) by using some ideas developed in [18,29–31,23].

The above works motivate us to consider (1.4) with locally distributed nonlinear damping, that is to consider the problem (1.1). Following an approach developed by Lasiecka anf Tataru [22], we construct a general energy decay formula, which are extensions of that in [16]. By the formulas we obtain several explicit energy decay rates for the solutions to (1.1). From the application's point of view, our results may provide some qualitative analysis and intuition for the researchers in other fields when they study the concrete models of form (1.1).

In paper [20] Komornick and Loreti proved the observability of the problem (1.1) without damping terms ($\rho_i = 0$) and with the boundary conditions replaced by

$$u_1 = \triangle u_1 = u_2 = 0, \quad x \in \partial\Omega, \ t > 0$$

by using a method related to nonharmonic analysis for vector-valued functions.

We mention here some related works concerning the energy decay rate for the nonlinear wave equations. For the nonlinear damped wave equation with polynomial growth damping terms in zero, some energy decay rate estimates results were obtained in [32,26,19] and the references there in. On the other hand, for the nonlinear damped wave equations with no growth condition on the feedback at the origin, some energy decay rate estimates results were established in [22,23,9,10,5,24,25,12,7,8].

There is much literature concerned with energy decay rate estimate for related problems, for more recent results we refer the reader to [4,6,11,13,27,28] and the references there in.

The rest of our paper is organized as follows. In Section 2 we present some preliminaries and main results. Section 3 is devoted to the proof of the main results.

2. Preliminaries and main results

To state our main results we make some assumptions. Let Ω be a bounded open domain in $\mathbb{R}^n$ of class C^4.

(A1) Assume that $a(x) \in L^\infty(\Omega)$.
(A2) Assume that the functions $\rho_i(\cdot,\cdot) \in C(\bar{\Omega} \times \mathbb{R})$ and is increasing with respect to the second variable, $i = 1, 2$ and

$$b_1 w^2 \leqslant \rho_i(x, w)w \leqslant b_2 w^2, \quad \forall x \in \Omega \text{ if } |w| \geqslant 1, \tag{2.1}$$

where b_1, b_2 are positive constants with $0 < b_2 \leqslant b_1$.
(A3) Assume the boundary $\partial\Omega$ of Ω satisfies $(x - x_0)\nu \leq 0$, where x_0 is some point in $\mathbb{R}^n$, ν is the outward normal on $\partial\Omega$

Define a function $h(\cdot) : [0, \infty) \to (-\infty, \infty)$, which is concave, and satisfies $h(0) = 0$, $h' > 0$, and

$$h(w\rho_i(x, w)) \geqslant w^2 + \rho_i^2(x, w), \quad |w| \leqslant 1, \ i = 1, 2. \tag{2.2}$$

The above function h does exist under our assumption (A1). We can construct h as in the following way. Define two increasing functions g_1, and g_2 satisfying