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Hybridizing ant colony optimization with firefly algorithm for unconstrained optimization problems

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ABSTRACT

We propose a novel hybrid algorithm named ACO–FA, which integrate ant colony optimization (ACO) with firefly algorithm (FA) to solve unconstrained optimization problems. The proposed algorithm integrates the merits of both ACO and FA and it has two characteristic features. Firstly, the algorithm is initialized by a population of random ants that roam through the search space. During this roaming an evolution of these ants are performed by integrating ACO and FA, where FA works as a local search to refine the positions found by the ants. Secondly, the performance of FA is improved by reducing the randomization parameter so that it decreases gradually as the optima are approaching. Finally, the proposed algorithm ACO–FA is tested on several benchmark problems from the usual literature and the numerical results have demonstrated the superiority of the proposed algorithm for finding the global optimal solution.

1. Introduction

Optimization problems are of importance for the industrial as well as the scientific world in many applications. There are many optimization problems that present attributes, such as high nonlinearity and multimodality, the solution of this kind of problems is usually a complex task. Moreover, in many instances, complex optimization problems present noise and/or discontinuities which make traditional deterministic methods inefficient to find the global solutions. Meanwhile, global optimization methods based on meta-heuristics are robust alternatives to solve complex optimization problems and do not require any properties of the objective function have been developed.

Due to the computational drawbacks of existing numerical methods, researchers have to rely on meta-heuristic algorithms based on simulations to solve some complex optimization problems. A common feature in meta-heuristic algorithms is that they combine rules and randomness to imitate natural phenomena. These phenomena include the biological evolutionary process (e.g., the genetic algorithm (GA) [9] and the differential evolution (DE) [23]), animal behavior (e.g., particle swarm optimization (PSO) [11] and ant colony algorithm (ACA) [5]), and the physical annealing process (e.g., simulated annealing (SA) [12]). Over the last decades, many meta-heuristic algorithms and their improved algorithms have been successfully applied to various engineering optimization problems [14,26,17,21,13,22,18]. They have outperformed conventional numerical methods on providing better solutions for some difficult and complicated real-world optimization problems.

Among the existing meta-heuristic algorithms, a well-known algorithm is the ACO which is a stochastic search procedure based on observations of social behaviors of real insects or animals. The original algorithm of ACO is known as the ant system [4] which was proposed by Dorigo to solve the traveling salesman problem. Since then, some algorithms based on the ACO

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are presented, such as the ant colony system, MAX–MIN Ant System and the rank-based ant system [5]. These algorithms are all based on the idea of updating the pheromone information to search the shortest route. Currently, the ACO has also been applied to continuous problems, where interesting results are discovered [3,6,15,16].

Recently, hybridization is recognized to be an essential aspect of high performing algorithms. Pure algorithms cannot reach to an optimal solution in a reasonable time. Thus pure algorithms are almost always inferior to hybridizations. Therefore, some researchers recently started investigating the incorporation of ACO algorithm with other techniques such as hybrid ant algorithms combined with beam search [2], constraint propagation [15], scatter search [24], simulated annealing for solving open shop scheduling problem [7] and mutation genetic operator by introducing a genetic algorithm into ant colony system [10] among others [3,6,19,16,20].

A promising new meta-heuristic algorithm denoted as firefly algorithm, which is inspired by social behavior of firefly and the phenomenon of bioluminescent communication. There are two important issues in the firefly algorithm that are the variation of light intensity and formulation of attractiveness. Yang [25] simplified that the attractiveness of a firefly is determined by its brightness which in turn is associated with the objective function. In general, the attractiveness is proportional to their brightness. Furthermore, every member of the firefly swarm is characterized by its bright that can be directly expressed as an inverse of an objective function for a minimization problem.

In this paper we propose a novel hybrid algorithm named ACO–FA for solving the unconstrained optimization problems. The motivation for a new hybrid algorithm is to overcome the drawback of classical ant colony algorithm which is not suitable for continuous optimizations. This methodology consists of two phases. The first one employs the meta-heuristic search by ACO, where the groups of candidate values of the variables are constructed and each value in the group has its trial information. At each iteration of ACO the solutions are constructed using the trial information, while the other employs firefly algorithm to improve the solution quality of optimization problems. The proposed algorithm has several characteristic features. Firstly, the algorithm is initialized by a set of random ants which is roaming through the search space. During this roaming an evolution of these ants is performed by integrating ACO and FA, where FA works as a local search to refine the positions found by the ants. Secondly, the performance of FA is improved by reducing the randomization parameter so that it decreases gradually as the optima are approaching. Finally, the proposed algorithm ACO–FA is tested on several benchmark problems from the usual literature and the numerical results have demonstrated the superiority of the proposed approach for finding the global optimal solution.

The organization of the remaining paper is as follows. In Section 2 we describe some preliminaries on optimization problems. In Sections 3 and 4, ACO and FA are briefly introduced. In Section 5, hybridizing ant colony optimization with firefly algorithm, named ACO–FA, is proposed and explained in details. Experiments and discussions are presented in Section 6. Finally, we conclude the paper in Section 7.

2. Preliminaries

The general numerical unconstrained optimization problem can be defined as follows [1]:
find $\underline{x}$ such that

$$\min F(\underline{x}), \underline{x} = (x_1, x_2, \ldots, x_n) \in \Re^n, \tag{1}$$

where $\underline{x} \in \Omega \subseteq S$. The objective function F is defined on the search space $S \subseteq \Re^n$ and the set $\Omega \subseteq S$ defines the feasible region. Usually, the search space S is defined as an n-dimensional rectangle in $\Re^n$, domains of variables defined by their lower and upper bounds:

$$x_j^L \leqslant x_j \leqslant x_j^U, j = 1, 2, \ldots, n. \tag{2}$$

3. Ant colony optimization (ACO)

ACO makes use of agents, called ants, which mimic the behavior of real ants in how they manage to establish shortest-route paths from their colony to feeding sources and back [5]. Ants communicate information through pheromone trails, which influence which routes the ants follow, and eventually lead to a solution route.

ACO was initially designed to solve the Traveling Salesman Problem (TSP). In the TSP, a given set of n cities has to be visited exactly once and the tour ends in the initial city. We call $d_{ij}, i, j = 1, 2, \ldots, n$, the length of the path between cities i and j. In the case of Euclidean TSP, d_{ij} is the Euclidean distance between i and j (i.e., $d_{ij} = \|x_i - x_j\|$). The cities and routes between them can be represented as a connected graph (n, E), where n the set of towns and E is the set of edges between towns (a fully connected graph in the Euclidean TSP).

The ants move from one city to another following the pheromone trails on the edges. Let $\tau_{ij}(t)$ be the trail intensity on edge (i,j) at iteration t. Then, each ant $k, k = 1, 2, \ldots, m$, chooses the next city to visit depending on the intensity of the associated trail. When the ants have completed their city tours, the trail intensity is updated according to Eqs. (3) and (4):

$$\tau_{ij}(t+1) = \rho \tau_{ij}(t) + \nabla \tau_{ij}, \quad t = 1, 2, \ldots, T, \tag{3}$$