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Global stability of a stage-structured predator–prey system

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A B S T R A C T

An autonomous stage-structured predator–prey system (stage structure for both predator and prey) with discrete delay is studied in this paper. By using an iterative method, the global stability of the interior equilibrium point of the system is investigated. Our result shows that conditions which ensure the permanence of the system are enough to ensure the global stability of the system. The result not only improves but also complements some existing ones.

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1. Introduction

Throughout this paper, for a bounded continuous function g defined on R, let g^i and g^s be defined as

$$g^i = \inf_{t\in R} g(t), \quad g^s = \sup_{t\in R} g(t).$$

Recently, Chen et al. [1,2] studied the persistence and extinction property of the following stage-structured predator–prey system (stage structure for both predator and prey, respectively).

$$\begin{aligned}
\dot{x}_1(t) &= r_1(t)x_2(t) - d_{11}x_1(t) - r_1(t-\tau_1)e^{-d_{11}\tau_1}x_2(t-\tau_1),\\
\dot{x}_2(t) &= r_1(t-\tau_1)e^{-d_{11}\tau_1}x_2(t-\tau_1) - d_{12}x_2(t) - b_1(t)x_2^2(t) - c_1(t)x_2(t)y_2(t),\\
\dot{y}_1(t) &= r_2(t)y_2(t) - d_{22}y_1(t) - r_2(t-\tau_2)e^{-d_{22}\tau_2}y_2(t-\tau_2),\\
\dot{y}_2(t) &= r_2(t-\tau_2)e^{-d_{22}\tau_2}y_2(t-\tau_2) - d_{21}y_2(t) - b_2(t)y_2^2(t) + c_2(t)y_2(t)x_2(t),
\end{aligned} \tag{1.1}$$

where $x_1(t)$ and $x_2(t)$ denote the densities of the immature and mature prey species at time t, respectively; $y_1(t)$ and $y_2(t)$ represent the immature and mature population densities of predator species at time t, respectively; $r_i(t),\ b_i(t),\ c_i(t)(i=1,2)$ are all continuous functions bounded above and below by positive constants for all $t \geqslant 0$. $d_{ij},\ \tau_i,\ i,j=1,2$ are all positive constants. By introducing a new lemma and applying the standard comparison theorem, The authors in [1] obtained the following result:

Theorem A. *If the assumptions*

(H_1')

$$x_2^i > 0$$

and

(H_2')

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$$y_2^i > 0$$

hold, where $x_2^i = (r_1^i e^{-d_{11}\tau_1} - d_{12} - c_1^s y_2^s)(b_1^s)^{-1}, x_2^s = (r_1^s e^{-d_{11}\tau_1} - d_{12})(b_1^i)^{-1}, y_2^i = (r_2^i e^{-d_{22}\tau_2} - d_{21} + c_2^i x_2^i)(b_2^s)^{-1},\ y_2^s = (r_2^s e^{-d_{22}\tau_2} + c_2^s x_2^s - d_{21})(b_2^i)^{-1}$. *Then system (1.1) is permanent.*

However, to this day, still no scholars investigate the stability property of the system (1.1), which is one of the most important topics in the study of population dynamics. We mentioned here that by constructing a suitable Lyapunov functional, one could always obtain conditions which ensure the global stability of the system, generally speaking, the conditions are very complicated and not easy to verify [17]. This motivated us to consider a slightly more simple system, i.e., the autonomous case of system (1.1).

$$\begin{aligned}
\dot{x}_1(t) &= r_1 x_2(t) - d_{11}x_1(t) - r_1 e^{-d_{11}\tau_1} x_2(t-\tau_1),\\
\dot{x}_2(t) &= r_1 e^{-d_{11}\tau_1} x_2(t-\tau_1) - d_{12}x_2(t) - b_1 x_2^2(t) - c_1 x_2(t) y_2(t),\\
\dot{y}_1(t) &= r_2 y_2(t) - d_{22} y_1(t) - r_2 e^{-d_{22}\tau_2} y_2(t-\tau_2),\\
\dot{y}_2(t) &= r_2 e^{-d_{22}\tau_2} y_2(t-\tau_2) - d_{21} y_2(t) - b_2 y_2^2(t) + c_2 y_2(t) x_2(t),
\end{aligned} \tag{1.2}$$

where $x_i(t)$ and $y_i(t),\ i = 1,2$ have the same meaning as that of system (1.1); $r_i,\ b_i,\ c_i (i = 1,2), d_{ij},\ \tau_i,\ i,j = 1,2$ are all positive constants.

The initial conditions for system (1.2) take the form of

$$\begin{aligned}
&x_i(\theta) = \phi_i(\theta) \geqslant 0, \quad y_i(\theta) = \psi_i(\theta) \geqslant 0,\\
&\phi_i(0) > 0, \quad \psi_i(0) > 0,\ i = 1,2, \theta \in [-\tau, 0],
\end{aligned} \tag{1.3}$$

where $\tau = \max\{\tau_1, \tau_2\}$, $(\phi_1(\theta), \phi_2(\theta), \psi_1(\theta), \psi_2(\theta)) \in C([-\tau, 0], R_{+0}^4) \underset{=}{\text{def}} C_+$, the Banach space of continuous functions mapping the interval $[-\tau, 0]$ into R_{+0}^4, where we define

$$R_{+0}^4 = \{(x_1, x_2, x_3, x_4) : x_i \geqslant 0, i = 1,2,3,4\}.$$

For continuity of initial conditions, we require

$$x_1(0) = \int_{-\tau_1}^0 r_1 \phi_2(s) e^{d_{11}s} ds, \quad y_1(0) = \int_{-\tau_2}^0 r_2 \psi_2(s) e^{d_{22}s} ds. \tag{1.4}$$

Noting that the first and third equations of system (1.2) are equivalent to the following integrate form:

$$x_1(t) = \int_{t-\tau_1}^t r_1 e^{-d_{11}(t-s)} x_2(s) ds. \tag{1.5}$$

$$y_1(t) = \int_{t-\tau_2}^t r_2 e^{-d_{22}(t-s)} y_2(s) ds. \tag{1.6}$$

The asymptotic behaviors of $x_1(t)$ and $y_1(t)$ are depended on that of $x_2(t)$ and $y_2(t)$. Therefore, in this paper we just need to study the asymptotic behavior for the subsystem of system (1.2).

$$\begin{aligned}
\dot{x}_2(t) &= r_1 e^{-d_{11}\tau_1} x_2(t-\tau_1) - d_{12}x_2(t) - b_1 x_2^2(t) - c_1 x_2(t) y_2(t),\\
\dot{y}_2(t) &= r_2 e^{-d_{22}\tau_2} y_2(t-\tau_2) - d_{21} y_2(t) - b_2 y_2^2(t) + c_2 y_2(t) x_2(t).
\end{aligned} \tag{1.7}$$

From [2] we know that $x_2(t) \to 0$ as $t \to +\infty$ if $r_1 e^{-d_{11}\tau_1} \leqslant d_{12}$ holds. Since we are focus on the stability property of the positive equilibrium, for the rest of the paper, we assume that $r_1 e^{-d_{11}\tau_1} > d_{12}$ holds.

Following is the main result of this paper:

Theorem 1.1. *If the assumptions*

(A_1)

$$c_1 c_2 < b_1 b_2, \tag{1.8}$$

(A_2)

$$b_2\left(1 - \frac{c_1 c_2}{b_1 b_2}\right)\left(r_1 e^{-d_{11}\tau_1} - d_{12}\right) > c_1\left(r_2 e^{-d_{22}\tau_2} - d_{21}\right) \tag{1.9}$$

and

(A_3)

$$b_1\left(r_2 e^{-d_{22}\tau_2} - d_{21}\right) + c_2\left(r_1 e^{-d_{11}\tau_1} - d_{12}\right) > 0 \tag{1.10}$$

hold. Then the unique interior equilibrium $E^*(x_2^*, y_2^*)$ of system (1.7) is globally attractive, that is,

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