Applied Mathematics and Computation 223 (2013) 461–471

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Alternating direction method for structure-persevering finite element model updating problem ☆

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ARTICLE INFO

Keywords:
Finite element model updating
Structure connectivity
Convex programming
Alternating direction method

ABSTRACT

The problem of finding the optimal approximation to the discrete stiffness matrix modeled by the finite element method is considered in this paper. Desired properties of the updated matrix, including symmetry, positive semidefiniteness and structure connectivity, are imposed as side constraints. Besides these, the optimal approximate matrix should be the least-squares solution to the dynamics equation. To the best of the author's knowledge, the optimal matrix approximation problem containing all these constraints simultaneously has not been proposed in the literature earlier. Alternating direction method is first applied to this constrained minimization problem. Numerical examples are performed to illustrate the efficiency of the proposed method.

1. Introduction

In this paper, we consider the following optimal matrix approximation problem

$$\begin{aligned} &\min \frac{1}{2}\|K - K_a\|_F^2 \\ &s.t. \|KX_e - M_a X_e \Lambda_e\|_F^2 = \min, \\ &K = K^T, \quad K \geqslant 0, \\ &\text{sparse } (K) = \text{sparse } (K_a), \end{aligned} \tag{1}$$

where $M_a,\ K_a \in R^{n\times n},\ \Lambda_e = \text{diag}(\lambda_1^{(e)},\ldots,\lambda_m^{(e)}) \in R^{m\times m},\ X_e = [x_1^{(e)},\ldots,x_m^{(e)}] \in R^{n\times m}$ and sparse $(K) = \text{sparse}(K_a)$ means that the zero/nonzero pattern of K is the same with that of K_a.

Problem (1) arises typically in structural dynamics model updating problem [1–5,20]. Let M_a and K_a be the discrete mass matrix and stiffness matrix of a real-life structure modeled by the finite element method respectively. The diagonal elements of Λ_e are natural frequencies obtained by vibration tests and X_e consists of corresponding measured mode shapes. Owing to the complexity of real-life structures, however, the finite element model depends on the hypothesis of the geometry, boundary conditions and connectivity conditions of real-life structures and fails to reproduce the dynamic behaviors accurately. To find the smallest adjustment to the discrete stiffness matrix can be formulated as the objective function of problem (1). To ensure the physical significance of the updated result, the least-squares solution to the dynamics equation, the nonnegativeness of energy and the structural connectivity are imposed as side constraints, as shown in problem (1).

Special versions of problem (1) have been studied by many authors [6–20] under the assumption that the discrete mass matrix and the measured data are exact, most of which consider part of the constraints of problem (1). In addition, in

☆ Supported by the NUAA Fundamental Research Funds No. NZ2013212.
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0096-3003/$ - see front matter
http://dx.doi.org/10.1016/j.amc.2013.08.031

consideration of measured error, the problem of processing incomplete, noisy measurements has stimulated some research efforts [15,19–23].

One of the typical approaches for dealing with this class of problems is the matrix decomposition methods [9,19,20], including the QR factorization, the singular value decomposition (SVD), the generalized singular value decomposition (GSVD), and the canonical correlation decomposition (CCD). In view of the sparsity on the updated matrix, we apply the alternating direction method (ADM) to problem (1) which was proposed by Gabay and Mercier [24] to solve separable convex programming. In recent years, alternating direction method has received extensive attention for its applications, including convex programming [25,26], variational inequalities [27], semidefinite programming [28] and image processing [29]. Methods and applications for solving systems of linear and nonlinear matrix equations which are closely related to those in this paper can be found in [30–32].

Throughout this paper, the following notations will be used. For $A, B \in R^{n\times m}$, $\langle A, B\rangle = tr(A^TB)$. S^n_+ is the set of all $n \times n$ symmetric positive semidefinite matrices. e_i is the ith column of the identity matrix I and P is an appropriate permutation matrix defined by $P = [e_{i_1}, e_{i_2}, \ldots, e_{i_n}]$, where $(i_1, i_2, \ldots, i_n)$ is a permutation of $(1, 2, \ldots, n)$. This paper is organized as follows. In Section 2, the original alternating direction method is briefly reviewed. In Section 3, two subproblems of problem (1) are studied and the alternating direction method for the equivalent form of problem (1) is proposed. In Section 4, two numerical experiments are performed to illustrate the efficiency of the proposed method. Conclusions are drawn in Section 5.

2. Preliminaries

In this section, we review ADM [24] briefly for completeness.

Let $G_1 : R^s \to (-\infty, +\infty]$ and $G_2 : R^t \to (-\infty, +\infty]$ be closed proper convex functions. A is a $t \times s$ matrix, and C_1 and C_2 are nonempty closed convex subsets of R^s and R^t respectively. Consider the following problem

$$\begin{aligned} &\min G_1(y) + G_2(z) \\ &s.t.\ Ay - z = 0, \\ &y \in C_1, \quad z \in C_2. \end{aligned} \tag{2}$$

Let $\lambda \in R^t$ be the Lagrange multiplier vector and r be a positive parameter which penalizes for the violation of the constraint. The augmented Lagrangian of problem (2) is

$$L_r(y, z, \lambda) = G_1(y) + G_2(z) + \langle \lambda, Ay - z\rangle + \frac{r}{2}\|Ay - z\|_2^2. \tag{3}$$

Given $y^{(k)}, z^{(k)}$ and $\lambda^{(k)}$, the iteration scheme of ADM may be described as

$$y^{(k+1)} = \underset{y\in C_1}{\operatorname{argmin}} \left\{G_1(y) + \langle \lambda^{(k)}, Ay\rangle + \frac{r}{2}\|Ay - z^{(k)}\|_2^2\right\}, \tag{4}$$

$$z^{(k+1)} = \underset{z\in C_2}{\operatorname{argmin}} \left\{G_2(z) - \langle \lambda^{(k)}, z\rangle + \frac{r}{2}\|Ay^{(k+1)} - z\|_2^2\right\}, \tag{5}$$

$$\lambda^{(k+1)} = \lambda^{(k)} + r[Ay^{(k+1)} - z^{(k+1)}]. \tag{6}$$

Noticing that the objective function of problem (2) is separable, ADM consisted of (4)–(6) is in fact a decomposition algorithm. The efficiency of this method depends on the solutions of two subproblems defined by (4) and (5). If $rank(A) = s$, the minimums of (4) and (5) are always uniquely attained. Hence, ADM is well defined for problem (2). For matrices, if we construct a vector consisted of all the column vectors of a matrix in sequence, the Frobenius norm of a matrix is the same as the 2-norm of a vector, and so is the inner product of vector pair and that of matrix pair. Hence, the vector type alternating direction method can be extended directly to the matrix type alternating direction method.

3. ADM for problem (1)

Necessary and sufficient conditions satisfying with the least-squares and positive semidefinite constraints in problem (1) are drawn in [20]. But when the sparsity constraint is appended, it is difficult to give the conditions for guaranteeing the nonemptiness of the feasible region of problem (1), denoted by D. Throughout this section, we make the assumption that D is nonempty.

3.1. Algorithm

To fit the form of problem (2), we define

$$S_1 = \left\{K \in R^{n\times n} | \|KX_e - M_aX_e\Lambda_e\|_F^2 = \min, \text{sparse}(K) = \text{sparse}(K_a)\right\}, \tag{7}$$

$$S_2 = \left\{K \in R^{n\times n} | K^T = K,\ K \geqslant 0\right\}. \tag{8}$$