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Hopf bifurcation analysis and amplitude control of the modified Lorenz system

Xuedi Wang *, Lianwang Deng, Wenli Zhang

Nonlinear Scientific Research Center, Faculty of Science, Jiangsu University, Zhenjiang, Jiangsu 212013, PR China

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ABSTRACT

This paper is concerned with the Hopf bifurcation analysis and amplitude control of the modified Lorenz system. The Hopf bifurcation of system is investigated by utilizing the Hopf bifurcation theory and the center manifold theorem firstly. Then the direction and stability of limit cycle emerging from Hopf bifurcation are determined by the designed controller. Moreover, the amplitude of limit cycle emerging from Hopf bifurcation is controlled by a nonlinear feedback controller. Finally, numerical simulations are given to verify theoretical analysis.

1. Introduction

Hopf bifurcation analysis of nonlinear dynamical system has been paid close attention widely over the last decade. Several methods for Hopf bifurcation analysis have been presented recently. In [1] the bifurcation control problem was analyzed by using state feedback and the stability of the limit cycle emerging from Hopf bifurcation was determined by its characteristic exponents, Kang and others [2,3] investigated the bifurcation problem by using normal forms and invariants. The center manifold theorem was used in [4] to study the system dynamics on a two dimensional manifold. They decompose the original system into two subsystems: a two-dimensional, non-hyperbolic system and a n-2 dimensional, hyperbolic one, then they expressed the hyperbolic part in the Brunovski form. In general, Hopf bifurcation control is mean to designing a controller what can change the bifurcation behavior of the original system. Controller can be designed to shift the location of bifurcation point, to change the direction and stability of Hopf bifurcation, etc. In particular, a typical objective is to monitor the amplitude of limit cycle. On the one hand, we can decrease amplitude to restrain the vibration behavior of system; on the other hand, we can obtain a desirable effect via increasing amplitude. Therefore, Hopf bifurcation control and amplitude control with various objectives have been implemented in mechanics, biology, and economy [5–9].

In this paper, we consider a modified Lorenz system as follows:

$$\begin{cases} \dot{x} = -ay, \\ \dot{y} = ax - exz, \\ \dot{z} = xy - bz, \end{cases} \tag{1}$$

where a, b and e are all positive real parameters. When $a = 2$, $b = 1$, $e = 1$, the Lyapunov exponent is 0.0022, −0.2009, −0.8014 respectively, therefore the system (1) is still a chaotic system (Fig. 1). In this paper we will investigate the Hopf bifurcation analysis and the amplitude control of system (1) by using different controllers.

* Corresponding author.
E-mail address: wxd959@ujs.edu.cnl (X. Wang).

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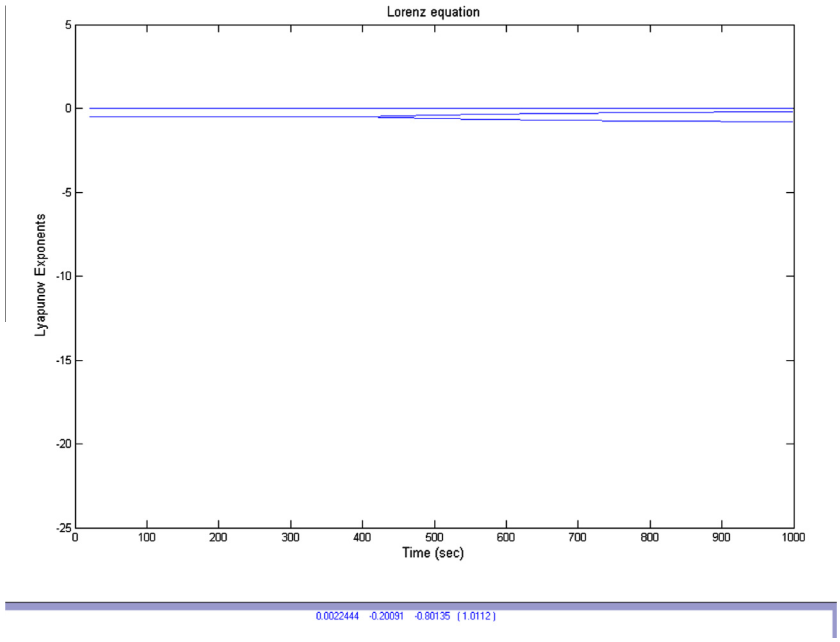

Fig. 1. The Lyapunov exponent of modified Lorenz system (1): $0.0022, -0.2009, -0.8014$.

The rest of this paper is organized as follows. In Section 2, the Hopf bifurcation control of the modified Lorenz system with controller is studied. In Section 3, the amplitude of limit cycle of the origin system and controlled system is analyzed. In Section 4, numerical simulations are given. Finally, a conclusion is drawn in Section 5.

2. Hopf bifurcation analysis

2.1. Preliminaries

Four essential theorems are included in this section. The center manifold theorem reduces the dimension of the system to two dimensions, and the Hopf bifurcation theorem establishes conditions to determine the direction and stability of the limit cycle emerging from Hopf bifurcation. In addition, the Theorems 3 and 4 give results about Hopf bifurcation control in different conditions.

Theorem 1 (The local Center Manifold Theorem, Perko [4]). *Let $f \in \Omega^r(\upsilon)$, where υ is an open subset of R^n containing the origin and $r \geqslant 1$. Suppose that $f(0) = 0$ and $Df(0)$ has c eigenvalues with zero real parts and s eigenvalues with negative real parts, where $c + s = n$. Then the system $\dot{\xi} = f(\xi)$ can be written in diagonal form*

$$\dot{x} = Ax + F_1(x, y), \quad \dot{y} = Bx + F_2(x, y),$$

where $(x, y) \in R^c \times R^s, A \in R^{c\times c}$ has c eigenvalues with zero real parts, $B \in R^{s\times s}$ has s eigenvalues with negative real parts, $F_i(0) = 0$ and $DF_i(0) = 0$, for $i = 1, 2$. Furthermore, there exists a c-dimensional invariant center manifold $W^c_{loc}(0)$ tangent to the center eigenspace E^c at the origin, where $W^c_{loc}(0) = \{(x, y) \in R^c \times R^s | y = h(x) \text{ for } x \in N_\delta(0)\}$, $N_\delta(0)$ is a neighborhood of the origin, with radius δ, and $h \in \Omega^r(N_\delta(0))$ satisfies

$$Dh(x)(Ax + F_1(x, h(x))) - Bh(x) - F_2(x, h(x)) = 0.$$

Finally, the flow on the center manifold is defined by the differential equations

$$\dot{x} = Ax + F_1(x, h(x)), \quad \text{for } x \in N_\delta(0).$$

Theorem 2 (Hopf Bifurcation Theorem, Guckenheimer and Holmes [10]). *Suppose that the system*

$$\dot{x} = f(x, \mu), \; x \in R^n, \; \mu \in R,$$

has an equilibrium point (x_0, μ_0) such that

(A1) *$D_x f(x_0, \mu_0)$ has a simple pair of pure imaginary eigenvalues and no other eigenvalues with zero real parts.*
(A2) *Let $\lambda_1(\mu)$, $\lambda_2(\mu)$ be the eigenvalues of $D_x f(x_0, \mu_0)$ which are imaginary at $\mu = \mu_0$, such that* $\mathrm{Re}\{\frac{\partial\lambda(\mu)}{\partial\mu}|_{\mu=\mu_0}\} = d \neq 0$.