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Upper semicontinuity of random attractors for stochastic three-component reversible Gray–Scott system ☆

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ABSTRACT

We consider the upper semicontinuity of the global random attractors for the stochastic three-component reversible Gray–Scott system on unbounded domains when the intensity of the noise converges to zero.

1. Introduction

The paper is devoted to concerning with the upper- semicontinuity of the random attractors for the stochastic three-component reversible Gray–Scott system on unbounded domains with multiplicative noise. Given a small positive parameter σ, consider the following stochastically perturbed system:

$$\begin{aligned} \frac{\partial \tilde{u}}{\partial t} &= d_1 \triangle \tilde{u} - (F+k)\tilde{u} + \tilde{u}^2\tilde{v} - G\tilde{u}^3 + N\tilde{w} + f_1(x) + \sigma \tilde{u} \circ \frac{d\omega}{dt}, \\ \frac{\partial \tilde{v}}{\partial t} &= d_2 \triangle \tilde{v} - F\tilde{v} - \tilde{u}^2\tilde{v} + G\tilde{u}^3 + f_2(x) + \sigma \tilde{v} \circ \frac{d\omega}{dt}, \\ \frac{\partial \tilde{w}}{\partial t} &= d_3 \triangle \tilde{w} + k\tilde{u} - (F+N)\tilde{w} + f_3(x) + \sigma \tilde{w} \circ \frac{d\omega}{dt}, \end{aligned} \tag{1.1}$$

with initial data

$$\tilde{u}(0,x) = \tilde{u}_0(x), \quad \tilde{v}(0,x) = \tilde{v}_0(x), \quad \tilde{w}(0,x) = \tilde{w}_0(x), \quad x \in \mathbb{R}^n, \tag{1.2}$$

where $\tilde{u} = \tilde{u}(x,t), \tilde{v} = \tilde{v}(x,t), \tilde{w} = \tilde{w}(x,t)$ are real-valued functions on $\mathbb{R}^n \times [0,\infty); f_i$ $(i = 1,2,3)$ are nonlinear functions satisfying certain conditions; all the parameters are arbitrarily given positive constants; ω is a two-sided real-valued Wiener process on a probability space $(\Omega, \mathcal{F}, \mathbb{P}), \Omega = \{\omega \in C(\mathbb{R},\mathbb{R}) : \omega(0) = 0\}$, the Borel sigma-algebra $\mathcal{F}$ on Ω is generated by the compact open topology (see [1]), $\mathbb{P}$ is the corresponding Wiener measure on $\mathcal{F}$ and $\circ$ denotes the Stratonovich sense of the stochastic term.

Historically, when $\tilde{w} = 0, G = 0, f_1 = f_3 = 0, f_2 = F$ and there are not random terms ($\sigma = 0$), system (1.1) reduces to the two-component Gray–Scott system which signified one of the Brussels school led by the renowned physical chemist and

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Nobel Prize laureate (1977), Ilya Prigogine, which originated from describing an isothermal, cubic autocatalytic, continuously fed and diffusive reactions of two chemicals (see e.g. [7–10]). The three-component reversible Gray–Scott model was firstly introduced by Mahara et al., which is based on the scheme of two reversible chemical or biochemical reactions [11]. Then in [2], You took some nondimensional transformations, the three-component reversible system was reduced to the system (1.1) without random forces. In [2], You considered the existence of global attractor for the system (1.1) with Neumann boundary condition on a bounded domain of space dimension $n \leqslant 3$ by the method of the re-scaling and grouping estimation. In [21], the uniform attractor of a non-autonomous three-component reversible Gray–Scott system was established.

Stochastic differential equations of this type arise from many chemical or biochemical systems when random spatio-temporal forcing is taken into consideration. These random perturbations are intrinsic effects in a variety of settings and spatial scales. They may be most obviously influential at the microscopic and smaller scales but indirectly they play an important role in macroscopic phenomena (see e.g. [12–14]). In [15–17], the influence of additive noise on Gray–Scott systems was discussed. Recently, Gu [18] gave the existence of a compact random attractor for stochastic three-component reversible Gray–Scott system with multiplicative white noise in a bounded domain of $\mathbb{R}^n$ $(n \le 3)$ when $f_1 = f_3 = 0, f_2 = F$ in system (1.1). The existence of random attractor for system 1.1 and 1.2 on unbounded domains was obtained in [19].

In this paper, we will study the limiting behavior of random attractors for the stochastically perturbed reversible three-component reaction–diffusion system (1.1) and (1.2) defined on $\mathbb{R}^{3n}$ when $\sigma \to 0$, and prove the upper semicontinuity of these perturbed random attractors. The main result reveals the robustness of the global asymptotic dynamics for such a class of perturbed stochastic reversible reaction–diffusion systems. The global dynamics considered here is novel and meaningful and it haven't been reported yet to the best of our knowledge. It is worth mentioning that when there is not random forcing $(\sigma = 0)$, the upper semicontinuity of global attractors as the inverse reaction rates (G, N) tend to $(0^+, 0^+)$ for the deterministic system was considered in [2] by an approach of transformative decomposition to avoid the singularity factors. The result is different from ours. Here, the main difficulty is the non-compactness of Sobolev embedding on $\mathbb{R}^n$. Based on the methodology of [6], we will overcome the obstacles by using uniform estimate for far-field values of functions lying in the perturbed random attractors. Actually, by a cut-off technique, we will show that the values of all functions in all perturbed random attractors are uniform convergent to zero when spatial variable approach infinity.

The paper is organized as follows. In Section 2, we recall some basic random attractors theory. Section 3 is devoted to the existence of the unique weak solution and define a continuous random dynamical system for system 1.1 and 1.2 in $\mathbb{H}$. In Section 4, we establish some uniform estimates of the solutions. Finally, we obtain the main result of upper- semicontinuity of the global random attractors in the last section.

We denote by $\|\cdot\|$ and $(\cdot,\cdot)$ the norm and inner product in $L^2(\mathbb{R}^n)$ or $\mathbb{H} = [L^2(\mathbb{R}^n)]^3$; let $\mathbb{V} = [L^4(\mathbb{R}^n)]^3$, $\mathbb{U} = [L^6(\mathbb{R}^n)]^3$, $\mathbb{E} = [H^1(\mathbb{R}^n)]^3$, $\|\cdot\|_{L^6}$, $\|\cdot\|_{L^4}$ and $\|\cdot\|_{\mathbb{V}}$, $\|\cdot\|_{\mathbb{U}}$ denote the norm in $L^4(\mathbb{R}^n)$, $L^6(\mathbb{R}^n)$ and $\mathbb{V}, \mathbb{U}$.

2. Preliminaries

In this section, we recall some basic concepts related to random attractors for random dynamical systems. We refer the reader to [1,4–6] for more details. Let $(X, \|\cdot\|_X)$ be a separable Hilbert space with Borel sigma-algebra $\mathcal{B}(X)$, and let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space.

Definition 2.1. $(\Omega, \mathcal{F}, \mathbb{P}, (\theta_t)_{t\in\mathbb{R}})$ is called a metric dynamical system, if $\theta : \mathbb{R} \times \Omega \mapsto \Omega$ is $(\mathcal{B}(\mathbb{R}) \times \mathcal{F}, \mathcal{F})$-measurable, θ_0 is the identity on Ω, $\theta_{s+t} = \theta_t\theta_s$ for all $s, t \in \mathbb{R}$ and $\theta_t\mathbb{P} = \mathbb{P}$ for all $t \in \mathbb{R}$.

Definition 2.2. A stochastic process $\{\varphi(t,\omega)\}_{t\geqslant 0,\omega\in\Omega}$ is a continuous random dynamical system (RDS) over $(\Omega, \mathcal{F}, \mathbb{P}, (\theta_t)_{t\in\mathbb{R}})$ if φ is $(\mathcal{B}[0,\infty) \times \mathcal{F} \times \mathcal{B}(X), \mathcal{B}(X))$-measurable, and for all $\omega \in \Omega$,

(i) The mapping $\varphi(t,\omega) : X \mapsto X, x \mapsto \varphi(t,\omega)x$ is continuous for every $t \geqslant 0$,
(ii) $\varphi(0,\omega)$ is the identity on X,
(iii) (cocycle property) $\varphi(s+t,\omega) = \varphi(t,\theta_s\omega)\varphi(s,\omega)$ for all $s, t \geqslant 0$.

Definition 2.3.

(i) A set-valued mapping $\omega \mapsto B(\omega) \subset X$ (we may write it as $B(\omega)$ for short) is said to be a random set if the mapping $\omega \mapsto \mathrm{dist}_{X(x,\, B(\omega))}$ is measurable for any $x \in X$.
(ii) A random set $B(\omega)$ is said to be bounded if there exist $x_0 \in X$ and a random variable $r(\omega) > 0$ such that $B(\omega) \subset \{x \in X : \|x - x_0\|_X \leqslant r(\omega), x_0 \in X\}$ for all $\omega \in \Omega$.
(iii) A random set $B(\omega)$ is called a compact random set if $B(\omega)$ is compact for all $\omega \in \Omega$.
(iv) A random bounded set $B(\omega) \subset X$ is called tempered with respect to $(\theta_t)_{t\in\mathbb{R}}$ if for a.e. $\omega \in \Omega$ $\lim_{t\to+\infty} e^{-\gamma t} \sup_{x\in B(\theta_{-t}\omega)} \|x\|_X = 0$ for all $\gamma > 0$. A random variable $\omega \mapsto r(\omega) \in \mathbb{R}$ is said to be tempered with respect to $(\theta_t)_{t\in\mathbb{R}}$ if for a.e. $\omega \in \Omega$, $\lim_{t\to+\infty} e^{-\gamma t} \sup_{t\in\mathbb{R}} |r(\theta_{-t}\omega)| = 0$ for all $\gamma > 0$.