Applied Mathematics and Computation 225 (2013) 541–549

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Asymptotic behaviors of stochastic Cohen–Grossberg neural networks with mixed time-delays

Li Wan [a,*], Qinghua Zhou [b]

[a] School of Mathematics and Computers, Wuhan Textile University, Wuhan 430073, China
[b] Department of Mathematics, Zhaoqing University, Zhaoqing 526061, China

ARTICLE INFO

Keywords:
Cohen–Grossberg neural networks
Mixed time-delays
Asymptotic stability
Ultimate boundedness

ABSTRACT

Asymptotic behaviors of stochastic Cohen–Grossberg neural networks with mixed time-delays are investigated, where the mixed time-delays comprise both the discrete time-varying delays and the distributed time-delays. The theory of stochastic functional differential equations is applied to establish two sets of novel criteria on asymptotic stability and ultimate boundedness. Finally, two examples are given to illustrate our theoretical results and to indicate that two sets of criteria do not include each other.

1. Introduction

Cohen–Grossberg neural networks was proposed by Cohen and Grossberg in 1983, which includes Hopfield neural networks, cellular neural networks and bidirectional associative memory neural networks as its special cases [1]. In both the biological and artificial neural networks, the interactions between neurons are generally asynchronous, which give rise to the inevitable signal transmission delays. Also, in electronic implementation of analog neural networks, time-delay is usually time-varying due to the finite switching speed of amplifiers. Note that continuously distributed delays have gained particular attention, since a neural network usually has a spatial nature due to the presence of an amount of parallel pathways of a variety of axon sizes and lengths. Many authors have paid much attention to the research of delayed Cohen–Grossberg neural networks and made some progress [2–15].

Recently, it has been well recognized that stochastic disturbances are ubiquitous and inevitable in various systems, ranging from electronic implementations to biochemical systems, which are mainly caused by thermal noise, environmental fluctuations as well as different orders of ongoing events in the overall systems [16,17]. Therefore, many authors have paid much attention to the research of stochastic neural networks and obtained some interesting results. Some sufficient criteria on the stability of uncertain stochastic neural networks were derived in [18–20]; Almost sure exponential stability of stochastic neural networks was discussed in [21–25]; In [26–32], mean square exponential stability and pth moment exponential stability of stochastic neural networks were investigated; Some sufficient criteria on the exponential stability for impulsive stochastic neural networks were established in [33–36]; In [37], the stability of discrete-time stochastic neural networks was analyzed; Exponential stability of stochastic neural networks with Markovian jump parameters is investigated in [38–40]. These references mainly considered the stability of equilibrium point of stochastic neural networks.

What do we study to understand asymptotic behaviors of stochastic neural networks when the equilibrium point dose not exist? Boundedness is also one of foundational concepts of dynamical systems. Recently, the results on ultimate boundedness of delayed neural networks have been reported. Some sufficient criteria on boundedness were derived in [41,42]; In [43], the globally robust ultimate boundedness of integro–differential neural networks with uncertainties and varying delays

* Corresponding author.
E-mail addresses: wanlinju@yahoo.com.cn, wanlinju@aliyun.com (L. Wan), zqhmath@yahoo.com.cn (Q. Zhou).

0096-3003/$ - see front matter © 2013 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.amc.2013.10.002

were studied; Some sufficient criteria on the ultimate boundedness of neural networks with both varying and unbounded delays were derived in [44]; In [45,46], a series of criteria on the boundedness, global exponential stability and the existence of periodic solution for non-autonomous recurrent neural networks were established; Ultimate boundedness of stochastic Hopfield neural networks were discussed in [47,48]. To the best of our knowledge, for stochastic neural networks with mixed time-delays, there are few published results on the ultimate boundedness and asymptotic stability. Motivated by the above discussions, the objective of this paper is to study ultimate boundedness and asymptotic stability of the stochastic Cohen–Grossberg neural networks with mixed time-delays.

The left paper is organized as follows, some preliminaries are in Section 2, main results are presented in Section 3, numerical examples are given in Section 4 and conclusions are drawn in Section 5.

2. Preliminaries

Consider the following stochastic Cohen–Grossberg neural networks with mixed time-delays

$$dx(t) = d(x(t))\left[-c(x(t)) + Af(x(t)) + Bf(x(t-\tau(t))) + D\int_{t-\tau(t)}^{t} g(x(s))ds + J\right]dt + [\sigma_1 x(t) + \sigma_2 x(t-\tau(t))]dw(t), \tag{2.1}$$

where $x = (x_1, \ldots, x_n)^T$ is the state vector, $A = (a_{ij})_{n\times n}, B = (b_{ij})_{n\times n}$ and $D = (d_{ij})_{n\times n}$ represent the connection weight matrix; $d(x(t)) = diag(d_1(x_1(t)), \ldots, d_n(x_n(t)))$ presents an amplification function, $c(x(t)) = (c_1(x_1(t)), \ldots, c_n(x_n(t)))^T$ presents an appropriately behavior function; $J = (J_1, \ldots, J_n)^T$ denotes the external bias; $\sigma_1, \sigma_2 \in R^{n\times n}$ are the diffusion coefficient matrices; $f(x(t)) = (f_1(x_1(t)), \ldots, f_n(x_n(t)))^T$ and $g(x(t)) = (g_1(x_1(t)), \ldots, g_n(x_n(t)))^T$ denote activation functions; $w(t)$ is one dimensional Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, P)$ with a natural filtration $\{\mathcal{F}_t\}_{t\geqslant 0}$ generated by $\{w(s) : 0 \leqslant s \leqslant t\}$; there exists a positive constant τ such that the transmission delay $\tau(t)$ satisfies $0 \leqslant \tau(t) \leqslant \tau$.

The initial conditions are given in the form:

$$x(s) = \xi(s), \quad -\tau \leqslant s \leqslant 0, \quad j = 1, \ldots, n,$$

where $\xi(s) = (\xi_1(s), \ldots, \xi_n(s))^T$ is $C([-\tau, 0]; R^n)$-valued function and $\mathcal{F}_0$-measurable R^n-valued random variable satisfying $\|\xi\|_\tau^2 = \sup_{-\tau\leqslant s\leqslant 0} E\|\xi(s)\|^2 < \infty$, $\|\cdot\|$ is the Euclidean norm and $C([-\tau, 0]; R^n)$ is the space of all continuous R^n-valued functions defined on $[-\tau, 0]$.

Let $F(x_t, t) = d(x(t))\left[-c(x(t)) + Af(x(t)) + Bf(x(t-\tau(t))) + D\int_{t-\tau(t)}^{t} g(x(s))ds + J\right], G(x_t, t) = \sigma_1 x(t) + \sigma_2 x(t-\tau(t))$, where

$$x_t = \{x(t+\theta) : -\tau \leqslant \theta \leqslant 0, t \geqslant 0\} = \varphi(\theta). \tag{2.2}$$

Then system (2.1) can be written by

$$dx(t) = F(x_t, t)dt + G(x_t, t)dw(t). \tag{2.3}$$

Throughout this paper, the following assumptions will be considered.

(A_1) There exist constants l_i^-, l_i^+, m_i^- and m_i^+ such that

$$l_i^- \leqslant \frac{f_i(x) - f_i(y)}{x - y} \leqslant l_i^+, \quad m_i^- \leqslant \frac{g_i(x) - g_i(y)}{x - y} \leqslant m_i^+, \quad \forall x, \quad y \in R.$$

(A_2) There exist matrices $\bar{d} = diag\{\bar{d}_1, \ldots, \bar{d}_n\} > 0$ and $\underline{d} = diag\{\underline{d}_1, \ldots, \underline{d}_n\} > 0$ such that $0 < \underline{d}_i \leqslant d_i(x) \leqslant \overline{d_i}$, for $x \in R, i = 1, 2, \ldots, n$.

(A_3) There exist constant v and matrix $\delta = diag\{\delta_1, \ldots, \delta_n\} > 0$ such that

$$\dot{\tau}(t) \leqslant v < 1, \quad x_i(t)c_i(x_i(t)) \geqslant \delta_i x_i^2(t), \quad i = 1, \ldots, n.$$

Remark 1. It follows from $(A_1), (A_2)$ and [49] that system (2.1) has a global solution on $t \geqslant 0$ and $F(x_t, t)$ and $G(x_t, t)$ satisfy the local Lipschitz condition in [50].

Remark 2. Assumption (A_1) is less conservative than that of in [18,19], since the constants l_i^-, l_i^+, m_i^- and m_i^+ are allowed to be positive, negative numbers or zeros.

The notation $A > 0$ (respectively, $A \geqslant 0$) means that matrix A is symmetric positive definite (respectively, positive semi-definite). A^T denotes the transpose of the matrix A. $\lambda_{min}(A)$ represents the minimum eigenvalue of matrix A. $*$ means the symmetric terms, I denotes identity matrix. Denote L_1, L_2, M_1, M_2 by

$$L_1 = diag\{l_1^- l_1^+, \ldots, l_n^- l_n^+\}, \quad L_2 = diag\{l_1^- + l_1^+, \ldots, l_n^- + l_n^+\},$$

$$M_1 = diag\{m_1^- m_1^+, \ldots, m_n^- m_n^+\}, \quad M_2 = diag\{m_1^- + m_1^+, \ldots, m_n^- + m_n^+\}.$$