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Weak magnetic field solution of Schrödinger equation in two dimensions and continuous density of states

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ABSTRACT

Green function method is applied to the solution of Schrödinger equation in two-dimensions where there is a constant and nearly uniform magnetic field B_o applied perpendicularly and there are infinitesimally weak scatterers within the system. Electronic density of states $n(E)$ obtained from the average Green function g_o is a continuous function of energy E and not in the form of discrete Landau spikes even in the pure system with no disorder. Self consistent calculation of self-energy gives two possible scattering time values, one leading to resistance increase with B_o while the other causing decrease in resistance.

1. Introduction

Green function G_o related to partial differential operator (PDO) D_o, which is derived from Schrödinger partial differential equation (PDE), is used in obtaining the basic properties of a two-dimensional (2D) system. These properties include density of states (DOS) $n(E)$ and conductivity σ under the effect of a constant perpendicular magnetic field B_o. Difficulties may arise both from non-self adjointness of D_o and from its eigenvalue spectrum's having a discrete part as well as a continuous one [1]. Due to these difficulties the first Landau solution [2], by likening the problem to a harmonic oscillator, is invalid for weak magnetic fields B_o but its results have been carried to date in a wide variety of journal articles [3–5] and books [6–9]. It says that even the smallest B_o will remove the continuous spectrum and make the spectrum discrete, a statement not supported by the experiment so far [10–12] and some severe objections have been directed to it [13,14] saying that it needs to be abandoned in many situations. But all the attempts simply concentrated on somehow broadening the so called Landau levels and remained far from convincing. It is therefore necessary to make more radical changes in our way of look at the problem which can be made, for example, by including the continuous spectrum with a suitable weight as we did in our recent work on thin film case [15].

For this purpose we have to define a vector potential $\boldsymbol{A}$ giving exactly constant $\boldsymbol{B}$ or nearly constant one. Traditionally $A_\varphi = B_o r/2$ has been chosen [7,16] but this brings a divergent term $\sim r^2$ into D_o which is difficult to make converge by any choice of basis functions. We have chosen a vector potential $\boldsymbol{A}$,

$$A_\varphi = (B_o r/2)\exp(-\omega_o \tau_o r/a_o), \tag{1}$$

which gives, on performing the rotational $(\nabla \times \boldsymbol{A})_z = \frac{1}{r}\frac{\partial}{\partial r}(rA_\varphi) = (B_o r/2)(2/r - \omega_o \tau_o r^2/a_o)\exp(-\omega_o \tau_o r/a_o)$ a nearly constant $\boldsymbol{B} = B\hat{z}$,

$$B = B_o \exp(-\omega_o \tau_o r/a_o) - (\omega_o \tau_o r/2a_o)B_o \exp(-\omega_o \tau_o r/a_o). \tag{2}$$

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We can show that this B can be made as close as we wish to a constant B_o. Defining the cyclotron frequency $\omega_o = eB_o/m$, length scale $a_o \sim 10^3$ nm and system's relaxation time τ_o we can examine the entire problem in the weak field region $\omega_o\tau_o \ll 1$. Typically we can take $\tau_o = 3.3\times10^{-11}$ s for pure samples [17], and a field of $B_o = 10^{-3}T$ has $\omega_o = 1.76\times10^8$ s^{-1} so that the condition $\omega_o\tau_o = 1.76\times10^8\times3.3\times10^{-11} = 0.0058 \ll 1$ is satisfied. If we change r within the interval $0, a_o$ the exponential term in Eq. (2) falls off very slowly. At a distance $r = a_o$, at about the system boundary, its value is $\simeq 1 - 0.0058 = 0.9942$ and B is nearly constant within the system. In Appendix Eq. (A.22) we have developed an argument to choose $a_o = A^{1/10}$ so that the system boundary may be extended to infinity together with the system's area A. For weaker fields this constancy becomes more perfect. The prefactor in the second term on the right of Eq. (2) has the value $\omega_o\tau_o r/2a_o \leqslant 0.0029$ so, we can say that the produced magnetic field is composed of the first term on the right, $B = B_o\exp(-\omega_o\tau_o r/a_o)$. The fall off B with r can be made as slow as we wish by choosing B_o to be weak. Experimental work for $n(E)$ is mainly concentrated on determining it via some kind of activation energy measurements [10], magneto-capacitance experiments [11] or magnetization measurements [12]. Experimentalists however feel constrained to use the Landau levels concept, due to lack of a better theory, although they point out [12] that there is a constant background DOS especially at weak magnetic fields $B_o \leqslant 0.1T$, origin of which is not understood. One principal aim of this work is to bring explanation to the constant background DOS.

Experiments for magnetoresistance $R(B)$ show that [17,19–21] there are no MR oscillations for weak B_o, oscillations in the longitudinal resistivity $\rho_{xx} = \rho$ begin at high fields $B_o \geqslant 0.1T$ and estimating this range of values constitutes the second goal. The weak field theory developed here predicts no MR oscillations and agrees qualitatively with the experiment. To explain these experimental facts used to Landau model has to be changed for weak fields. In a recent work [15] I have solved the problem of finding G_o for the thin film case and now I apply this technique to 2D electron gas (2DEG) under the effect of a perpendicular magnetic field B_o.

In Section 2 I set out the model, obtain $G_o = G_o^c + G_o^d$ from basis functions u, v of D_o and its adjoint D_o^* respectively. Section 3 is about $n(E)$ and $R(B)$ calculations, in Section 4 they are compared with experiments and discussed, while Section 5 is for conclusions. Eigenfunctions and eigenvalues of D_o and D_o^* are described in a separate Appendix.

2. Model

Electron gas is in the infinite x, y plane where there are randomly distributed scatterers $V_b = \sum_i v(\boldsymbol{r} - \boldsymbol{R}_i)$ within the body and the constant magnetic field B_o points in the z direction. Using cylindrical coordinates and taking the vector potential given by Eq. (1), rearranged Schrödinger equation is $H_o\psi_o = (2mE/\hbar^2)\varepsilon_b V_b\psi_o$. Here the Hamiltonian $H_o = \{\frac{-1}{r}(\frac{\partial}{\partial r}r\frac{\partial}{\partial r}) - (\frac{\partial}{r\partial\varphi} + i\frac{eB_o r}{2\hbar}\exp(-\omega_o\tau_o r/a_o))^2\}$ is defined [9,16], e is the magnitude of the electronic charge, m is its mass, E is energy and i shows the imaginary number. The strength of scattering is made small by taking ε_b to be infinitesimal so that weak scattering results of conductivity theory applies [18]. We try to solve the modified Schrödinger equation

$$D_o\psi_o = \left(\frac{\partial^2}{\partial r^2} + \frac{\partial}{r\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2} + \frac{im\omega_o e^{-\frac{\omega_o\tau_o r}{a_o}}}{\hbar}\frac{\partial}{\partial\varphi} - \frac{m^2\omega_o^2 r^2 e^{-2\frac{\omega_o\tau_o r}{a_o}}}{4\hbar^2} + k_F^2 + i\varepsilon\right)\psi_o = \frac{-2m}{\hbar^2}\varepsilon_b V_b(r,\varphi)\psi_o, \tag{3}$$

in the vanishing volume scattering $\varepsilon_b \to 0$ limit, with $k_F^2 = \frac{2mE}{\hbar^2}$ where the operator D_o is defined as

$$D_o = \frac{\partial^2}{\partial r^2} + \frac{\partial}{r\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2} + \frac{im\omega_o}{\hbar}e^{-\omega_o\tau_o r/a_o}\frac{\partial}{\partial\varphi} - \frac{m^2\omega_o^2 r^2}{4\hbar^2}e^{-2\omega_o\tau_o r/a_o} + k_F^2 + i\varepsilon. \tag{4}$$

This operator has the same eigenfunctions as the original Schrödinger operator but its eigenvalues differ in units by $\hbar/2m$ times, in fact multiplying them by $2m/\hbar$ restores their unit to s^{-1}, as needed in self-energy calculations, which will be done at the end of the calculations.

Green function related to Eq. (3) satisfies

$$D_o G_o(r,\varphi;r',\varphi') = \delta(r-r')\delta(\varphi-\varphi'), \tag{5}$$

and serves in the solution as $\psi_o \sim \int G_o(r,\varphi;r',\varphi')\frac{2m}{\hbar^2}\varepsilon_b V_b(r',\varphi')\psi_o(r',\varphi')dr'd\varphi'$. Although the definition of G_o carries no trace of imperfections, solution ψ_o carries due to the term on the right of Eq. (3). So if we are to express G_o, ψ_o and V_b in a set of basis functions u, v, these functions should carry in them some randomness at least by trace amount, because if we tried to expand a random function, like ψ_o, in terms of a basis which had no randomness we would be in trouble. We assume that we can use a biorthogonal basis functions u, v where u are eigenfunctions of the operator D_o itself and v are those of the adjoint operator D_o^* with the common eigenvalue [25,26]. There are two possible choices for them, called the first choice and the second choice, and they are all explained in Appendix.

Using the biorthogonal basis given in Appendix, unperturbed Green function $G_o = G_o^c + G_o^d$ can be expanded in the first choice, as

$$G_o(r,\varphi;r',\varphi') = (1-\omega_o\tau_o)\cdot\sum_k\frac{u_k^o(r,\varphi)v_k^o(r',\varphi')}{k_F^2 - k^2 + i\varepsilon} + \omega_o\tau_o\cdot\sum_n\frac{u_n^o(r,\varphi)v_n^o(r',\varphi')}{k_F^2 - n\omega_o m/\hbar + i\varepsilon}. \tag{6}$$

Here τ_o is the system's relaxation time, unknown yet, due to the scattering term W on the right side of Eq. (3)