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Combined influence of velocity slip, temperature and concentration jump conditions on MHD peristaltic transport of a Carreau fluid in a non-uniform channel

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ABSTRACT

The effect of velocity slip, temperature and concentration jump conditions on the MHD peristaltic flow of a Carreau fluid in a non-uniform channel with heat and mass transfer is investigated. Using a perturbation technique, for small Weissenberg number, solutions are obtained for the non-linear differential equations with appropriate boundary conditions. The expressions for the stream function, velocity, temperature and concentration fields; and the heat transfer coefficient are obtained. The effects of the physical parameters on the velocity and temperature fields are presented through graphs, and are discussed. Also, the trapping phenomenon is analyzed. It is observed that the size of the trapping bolus increase with an increase in the velocity slip parameter β_1.

1. Introduction

Peristalsis occurs naturally while pumping physiological fluids from one place in the body to another place by means of a progressive wave of area contraction and expansion of an extensible tube propagating along the length of the tube. This mechanism frequently appears in organs such as ureters, intestines and the circulation of blood in small blood vessels such as arterioles, venules and capillaries. In the nuclear industry, a toxic liquid can be transported peristaltically in order to avoid contamination of the outside environment. Most of the physiological fluids including blood are observed to be non-Newtonian in nature. Several investigations on the peristaltic transport of non-Newtonian fluids under the assumptions of long wavelength (low Reynolds number) have been presented in the literature. A few investigations on this topic have been reported in [1–8].

Srivastava et al. [9] and Srivastava and Srivastava [10] studied peristaltic transport of Newtonian and non-Newtonian power-law fluids in uniform and non-uniform channels. Several studies are being made on the peristaltic motion in a channel with the effect of wall properties. Mittra and Prasad [11] analyzed the peristaltic motion of Newtonian fluid under the influence of flexible walls which were assumed to be either thin elastic plates or membranes. Pandey and Chaube [12] studied the effects of wall properties on peristaltic transport of a couple-stress fluid by Perturbation method. Muthu et al. [13] studied the influence of wall properties in the peristaltic motion of micropolar fluid. Radhakrishnamacharya and Srinivasulu [14] discussed the wall effects on the peristaltic transport in a uniform channel with heat transfer. The mathematical model for the MHD peristaltic motion of Johnson–Segalman fluid in a channel with compliant walls was analyzed by Hayat et al.

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[15]. The influence of slip, wall properties on MHD peristaltic transport of a Newtonian fluid with heat transfer and porous medium in a non-uniform channel has been investigated by Srinivas et al. [16].

Peristaltic flows with heat and mass transfer have many applications in the biomedical sciences. Bio-heat transfer processes include thermoregulation, metabolic heat generation, evaporation, convection, perfusion of blood flow, skin burning, fever and hypothermia in a living body system. Similarly, bio-mass transfer processes include the blood as oxygen carrier, equilibrium of oxygen in blood with inhaled air, diffusive oxygen transfer in a tissue, diffusive and ionic flows through membrane channels, drug delivery to local regions inside body, diffusion of gastric juice in the stomach. A few recent investigations on this topic have been reported by several authors [17–20].

Navier's condition [21] states that the existence of slip at the surface is linearly proportional to the shear stress at the surface. Fluids exhibiting the slip effect are important in polishing artificial heart valves and internal cavities. Investigations of the effects of slip on the peristaltic motion have been recently reported in [22–26].

A few attempts have been made to investigate the heat and mass transfer effects on the MHD peristaltic transport of non-Newtonian fluid by considering the wall properties. Hayat et al. [27,28] discussed the effects of heat and mass transfer on the peristaltic flow of MHD second grade fluid and Power-law fluid in a channel with flexible walls. Also, Hayat et al. [29,30] investigated the influence of wall properties, slip conditions on the MHD peristaltic flow of Maxwell fluid in a planar channel and Williamson fluid in a non-uniform channel with heat and mass transfer.

Further, it is speculated that the physiological fluids such as blood exhibits Newtonian and non-Newtonian behaviors during circulation in a living body. Thus it will be interesting to study the flow of a non-Newtonian fluid with the influence of wall properties and slip conditions. Hence, we choose the constitutive equation of the rheological model of Carreau fluid (for details see Bird et al. [31]). Also, Carreau model reduces to Newtonian model when $n = 1$ (or) $\Gamma = 0$. The model of Carreau fluid has been studied in channels and tubes [32–36].

In view of these applications, in the present study we investigate the influence of wall properties, heat and mass transfer on the peristaltic flow of a conducting Carreau fluid in a non-uniform channel using the velocity slip; temperature and concentration jump boundary conditions. The fluid flow is investigated in a fixed frame of reference under the assumption of long wavelength approximation. The perturbation solution for the velocity, stream function, temperature, coefficient of heat transfer and concentration are obtained in terms of small Weissenberg number. The effects of different physical parameters are obtained and their salient features are discussed through graphs.

2. Mathematical formulation

We consider the flow of an incompressible Carreau fluid in a non-uniform channel induced by sinusoidal waves propagating with a constant speed 'c'. The walls of the channel are flexible which are considered as a stretched membrane. A uniform magnetic field B_0 is applied along the y -axis. The lower and upper walls of the channel are maintained at constant temperatures T_0 and T_1, and at constant concentrations C_0 and C_1 respectively (see for details Fig. 1).

The wall deformation $\eta(x,t)$ due to the infinite train of peristaltic waves is represented by

$$y = \pm\eta(x,t) = \pm\left(d(x) + a\sin\frac{2\pi}{\lambda}(x - ct)\right), \tag{1}$$

where $d(x) = d + m'x, (m' \ll 1)$, a is the amplitude, λ is the wavelength, d is the mean half width of the channel, and t is the time and m' is the dimensional non-uniformity parameter of the channel.

The equations governing the MHD flow of an incompressible fluid in the absence of body force are

$$div\bar{V} = 0, \tag{2}$$

$$\rho\frac{d\bar{V}}{dt} = -PI + div\bar{\tau} + \bar{J} \times \bar{B}, \tag{3}$$

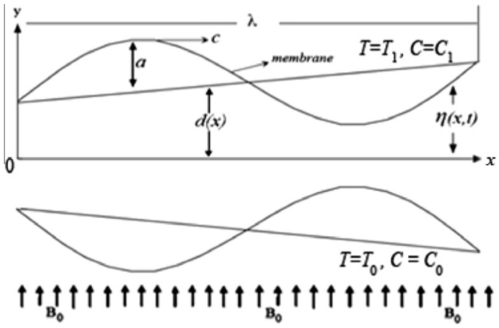

Fig. 1. Physical model.