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Approximate controllability of fractional nonlinear differential inclusions

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ABSTRACT

Fractional differential equations have wide applications in both physical and social sciences. This paper addresses the issue of approximate controllability for a class of fractional nonlinear differential inclusions in Banach spaces. A new set of sufficient conditions are formulated and proved for the approximate controllability of fractional nonlinear differential inclusions. In particular, the results are established with the assumption that the associated linear part of system is approximately controllable. Further, the result is extended to obtain the conditions for the solvability of controllability results for fractional inclusions with nonlocal conditions. Finally, an example is presented to illustrate the theory of the obtained result.

1. Introduction

Controllability is one of the primary concept in mathematical control theory. The concept of controllability plays an important role in both finite and infinite dimensional systems. Exact controllability for integer order dynamical systems have been studied by many authors [6,7,12,13,15–17,20,27]. Mahmudov [22] studied the exact null controllability for the abstract nonlinear evolution systems in Hilbert spaces by assuming controllability of the associated linear system. Klamka [18] derived a set of sufficient conditions for constrained local relative controllability for semilinear finite-dimensional dynamical control systems with multiple point delays in the control by using a generalized open mapping theorem. Abada [9] investigated the existence of integral solutions and extremal integral solutions for some nondensely defined impulsive semilinear functional differential inclusions and controllability results of the these equations in separable Banach spaces by using semigroup theory and fixed point technique. Exact controllability enables to steer the system to arbitrary final state while approximate controllability means that system can be steered to arbitrary small neighbourhood of final state. Moreover, approximate controllable systems are more prevalent and very often approximate controllability is completely adequate in applications. The approximate controllability of the first and second order dynamical systems has been studied in [10,11,19,26,28]. Approximate controllability for semilinear deterministic and stochastic control systems can be found in Mahmudov [21].

In recent years, fractional differential equations have attracted many physicists, mathematicians and engineers and notable contributions have been made to both theory and applications of fractional differential equations [8,23,35]. In fact, fractional differential equations are used for many mathematical models in wave propagation, electromagnetism, heat transfer, biology, signal processing, robotics, genetic algorithms, telecommunications and so on. Moreover, physical systems can be represented more accurately through fractional derivative formulation. On the other hand, the fractional differential inclu-

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sions arise in the mathematical modeling of certain problems in economics, optimal controls, etc., so the problem of existence of solutions of fractional differential inclusions has been studied by several authors for different kind of problems. (see [9,32,33] and references therein). It should be noted that it is generally difficult to realize the conditions of exact controllability for infinite-dimensional systems and thus the approximate controllability becomes a very important topic for dynamical systems [14]. It should be mentioned that there are only limited number of papers on the approximate controllability of the fractional nonlinear evolution systems [3,4]. Sakthivel et al. [24,25] studied the approximate controllability results for deterministic and stochastic fractional differential systems by using fixed point technique and fractional calculations. The controllability of fractional-order partial neutral functional integro differential inclusions with infinite delay has been studied in [34]. More recently, Surendra Kumar and Sukavanam [5] obtained a new set of sufficient conditions for the approximate controllability of a class of semilinear delay control systems of fractional order by using contraction principle and the Schauder fixed point theorem.

Upto now, approximate controllability problems for nonlinear fractional differential inclusions have not been considered in the literature, in order to fill this gap, this paper studies the approximate controllability of nonlinear fractional inclusions of the form

$$\begin{cases} {}^cD_t^q x(t) \in Ax(t) + F(t,x(t)) + Bu(t), \quad t \in J = [0,b],\ 0 < q < 1, \\ x(0) = x_0, \end{cases} \tag{1}$$

where the state $x(\cdot)$ takes the values in a Banach space X with norm $\|\cdot\|$; $b > 0$ is a finite number; ${}^cD_t^q$ denotes Caputo fractional derivative of order q; A is the infinitesimal generator of a C_0-semigroup $\{T(t), t \geqslant 0\}$ in the Banach space X; the control function $u(\cdot)$ is given in $L^2(J,U)$, U is a Banach space; B is a bounded linear operator from U into X; $F : J \times X \to 2^X \setminus \{\emptyset\}$ is a nonempty bounded, closed and convex multivalued map; x_0 is an element of X. The purpose of this paper is to show the approximate controllability of nonlinear differential inclusions of the form (1) in a Banach space under simple and fundamental assumptions on the system operators, in particular that the corresponding linear system is approximate controllable. The main objective of this paper is to obtain sufficient conditions for the approximate controllability of fractional order differential inclusions. To prove the result we use the techniques similar to that of [32] with suitable modifications. More precisely, the controllability problem can be converted into solvability problem of a functional operator equation in appropriate Banach spaces and fixed point theory is used to solve the problem.

2. Preliminaries

In this section, we state some definitions, notations and preliminary facts about fractional calculus and the multi-valued analysis [1,32].

Let $A : D(A) \to X$ be the infinitesimal generator of a strongly continuous semigroup $\{T(t)\}_{t \geqslant 0}$ and there exists a $M > 0$ such that $\sup_{t \in J}\|T(t)\| \leqslant M$. Let Y be another Banach space; $L_b(X,Y)$ denotes the space of bounded linear operators from X to Y. Let $\|f\|_{L^p(J,R^+)}$ denote the $L^p(J,R^+)$ norm of f whenever $f \in L^p(J,R^+)$ for some p with $1 \leqslant p < \infty$. Let $L^p(J,X)$ be the Banach space of functions $f : J \to X$ which are Bochner integrable normed by $\|f\|_{L^p(J,X)}$. We denote by $\mathcal{C}$, the Banach space $C(J,X)$ endowed with supnorm given by $\|x\|_{\mathcal{C}} \equiv \sup_{t \in J}\|x(t)\|$, for $x \in \mathcal{C}$.

Definition 2.1 [23]. The fractional integral of order β with the lower limit 0 for a function f is defined as

$$I^\beta f(t) = \frac{1}{\Gamma(\beta)} \int_0^t \frac{f(s)}{(t-s)^{1-\beta}} ds, \quad t > 0,\ \beta > 0,$$

provided the righthand side is pointwise defined on $[0,\infty)$, where $\Gamma(\cdot)$ is the gamma function.

Definition 2.2 [23]. The Caputo derivative of order β for a function $f : [0,\infty) \to R$ can be written as

$${}^cD^\beta f(t) = \frac{1}{\Gamma(n-\beta)} \int_0^t \frac{f^n(s)}{(t-s)^{\beta+1-n}} ds = I^{n-\beta} f^n(t), \quad t > 0,\ 0 \leqslant n-1 < \beta < n.$$

If f is an abstract function with values in X, then integrals which appears in the above definitions are taken in Bochner's sense.

A multivalued map $G : X \to 2^X \setminus \{\emptyset\}$ is convex (closed) valued if $G(x)$ is convex (closed) for all $x \in X$. G is bounded on bounded sets if $G(C) = \bigcup_{x \in C} G(x)$ is bounded in X, i.e., $\sup_{x \in C}\{\sup\{\|y\| : y \in G(x)\}\} < \infty$.

Definition 2.3. G is called upper semicontinuous (u.s.c.) on X if for each $x_0 \in X$, the set $G(x_0)$ is a nonempty closed subset of X, and if for each open set C of X containing $G(x_0)$, there exists an open neighbourhood V of x_0 such that $G(V) \subseteq C$.

Definition 2.4. G is called completely continuous if $G(C)$ is relatively compact for every bounded subset C of X.