



Monotone iterative solutions for nonlinear boundary value problems of fractional differential equation with deviating arguments [☆]



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ABSTRACT

By means of the monotone iterative technique and the method of lower and upper solutions, we consider the nonlinear boundary value problems with Riemann–Liouville fractional derivative and deviating arguments, introduce two well-defined monotone sequences of lower and upper solutions which converge uniformly to the actual solution of the problem, and then the existence result of solution for the problems are established. A numerical iterative scheme is introduced to obtain an accurate approximate solution for the problem. As an application, an example is presented to demonstrate the accuracy and efficiency of the new approach.

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1. Introduction

We are interested in the existence and uniqueness of solution of the following nonlinear boundary value problem of fractional order with deviating arguments and nonlinear boundary conditions (BVP)

$$\begin{cases} D^\mu u(t) = f(t, u(t), u(\theta(t))), & t \in (0, 1], \\ g(\tilde{u}(0), \tilde{u}(1)) = 0, \end{cases} \quad (1.1)$$

where $0 < \mu < 1$, $f \in C([0, 1] \times R \times R, R)$, $g \in C(R \times R, R)$, $\theta \in C([0, 1], [0, 1])$, $\tilde{u}(0) = t^{1-\mu}u(t)|_{t=0}$, $\tilde{u}(1) = t^{1-\mu}u(t)|_{t=1}$, and D^μ is the Riemann–Liouville fractional derivative of order μ .

It is well known that the differential equations with fractional order are generalization of ordinary differential equations to non-integer order, it occur more frequently in different research areas and engineering, such as physics, control of dynamical systems, chemistry etc. We also remark that several kinds of fractional derivatives were introduced to investigate the fractional differential equation, see for example [1,2] and references therein.

Roughly speaking, it is a difficult task to give exact solutions for fractional differential equations. Recently, there are a number of numerical and analytical techniques to concerned with such problems, for instance, the homotopy analysis method and the Adomian decomposition method have been discussed the fractional differential equations, such as [3,4,4,5]. On the other hand, very recently, the monotone iterative technique, combined with the method of lower and upper solutions were introduced to study the problems [6–9]. In [6,7], the authors used the method of lower and upper solutions to investigate the existence of solutions for a class of fractional initial value problems. In [8] the author considered fractional boundary value problem and proved the existence of solution. In [9] introduce two well-defined monotone sequences of lower and

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upper solutions which converge uniformly to the actual solution of the problem. Specially, here it is worth mentioning, that Wang [10] discuss the monotone iterative technique for boundary value problems of a nonlinear fractional differential equation with deviating arguments for BVP (1.1). However, the existence results in [10] mainly depend upon a restrictive condition, i.e.,

$$f(t, x, y) - f(t, \bar{x}, \bar{y}) \geq -M(x - \bar{x}).$$

It is a critical condition in order to discuss the monotone iterative sequences. It is, therefore, natural to ask whether similar results can be obtained if

$$f(t, x, y) - f(t, \bar{x}, \bar{y}) \geq -M(x - \bar{x}) - N(y - \bar{y}).$$

Which is of particular mathematical interest.

Being directly inspired by Wang [10], the purpose of this paper to study the nonlinear boundary value problems of fractional order with deviating arguments and nonlinear boundary conditions. We introduce a method based on lower and upper solutions to prove the existence of a unique solution. And also give an algorithm to construct two monotone sequences of lower and upper solutions. Moreover, the constructed sequences are proved to converge uniformly to the unique solution of the problem.

The paper is organized as follows: Preliminaries are in Section 2. Then in Section 3, we construct the monotone sequences of lower and upper solutions and prove their uniform convergence to the unique solution of the problem. Finally, in Section 4, we establish the numerical approach employed to obtain accurate numerical solution, and an example is presented to demonstrate the accuracy and efficiency of the new approach.

2. Preliminaries

In this sections, we introduce the definition of the lower and upper solutions and some Lemmas which will be needed in the next section.

For $0 < \mu < 1$, the Riemann–Liouville fractional derivative of order μ is defined by (see [1,2])

$$D^\mu h(t) = \frac{1}{\Gamma(1-\mu)} \frac{d}{dt} \int_0^t (t-s)^{-\mu} h(s) ds = \frac{d}{dt} I^{1-\mu} h(t),$$

here

$$\int_0^t (t-s)^{-\mu} h(s) ds = I^{1-\mu} h(t)$$

is Riemann–Liouville fractional integral of order $1 - \mu$ (see [1,2]).

Setting

$$C_{1-\mu}([0, 1]) = \{u \in C(0, 1) : t^{1-\mu}u \in C([0, 1])\}.$$

Definition 2.1. Let $u : C_{1-\mu}([0, 1]) \rightarrow R$ be locally Hölder continuous. We say that u is called a lower solution of BVP (1.1), if it satisfies

$$\begin{cases} D^\mu u(t) \leq f(t, u(t), u(\theta(t))), \\ g(\bar{u}(0), \bar{u}(1)) \leq 0 \end{cases}$$

and it is a upper solution of BVP (1.1). If the all above inequality are reversed.

Lemma 2.1. [12] Let $\varphi : R^+ \rightarrow R$ be locally Hölder continuous such that for any $t_1 \in (0, \infty)$. We have

$$\begin{cases} \varphi(t_1) = 0, \\ \varphi(t) \leq 0, \quad t \in [0, t_1]. \end{cases}$$

Then it follows that

$$D^\mu \varphi(t_1) \geq 0.$$

Lemma 2.2. Let $M_0 > 0, N_0 > 0$ be constants, $0 \leq \theta(t) \leq t, t \in [0, 1], p : C_{1-\mu}([0, 1]) \rightarrow R$ be locally Hölder continuous, and satisfies

$$\begin{cases} D^\mu p(t) + M_0 p(t) + N_0 p(\theta(t)) \geq 0, \quad t \in (0, 1], \\ \tilde{p}(0) = t^{1-\mu} p(t)|_{t=0} \geq 0. \end{cases} \quad (2.1)$$

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