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Stationary iteration methods for solving 3D electromagnetic scattering problems

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ABSTRACT

Generalized Chebyshev iteration (GCI) applied for solving linear equations with nonselfadjoint operators is considered. Sufficient conditions providing the convergence of iterations imposed on the domain of localization of the spectrum on the complex plane are obtained. A minimax problem for the determination of optimal complex iteration parameters is formulated. An algorithm of finding an optimal iteration parameter in the case of arbitrary location of the operator spectrum on the complex plane is constructed for the generalized simple iteration method. The results are applied to numerical solution of volume singular integral equations (VSIEs) associated with the problems of the mathematical theory of wave diffraction by 3D dielectric bodies. In particular, the domain of the spectrum location is described explicitly for low-frequency scattering problems and in the general case. The obtained results are discussed and recommendations concerning their applications are given.

1. Introduction

Analysis of the scattering of electromagnetic waves by three-dimensional inhomogeneous anisotropic dielectric structures is of crucial importance for studying various applied and theoretical issues. In the short-wave range, the asymptotical methods have proved to be the most efficient technique. In the resonance and low-frequency ranges, it is necessary to solve exact equations that describe the wave scattering; namely, the Maxwell equations with radiation conditions at infinity or volume singular integral equations (VSIEs) with respect to the wave field in the dielectric structure. Partial differential equations (PDEs) may seem to be more appropriate for numerical solution because their discretization results in a system of linear algebraic equations (SLAE) with a sparse matrix, while in the case of a VSIE this matrix is dense. However, when a scattering problem is considered, the solution must satisfy the condition at infinity; therefore, in order to provide the required accuracy the unknown wave field must be determined in a domain which is much larger than the actual dielectric scatterer. Finally, taking into account that the scattering problem is three-dimensional, we obtain a SLAE of extremely high size when PDEs are discretized and solved numerically by a finite-difference (FD) or a finite-element method (FEM). Application of approximate conditions at infinity often leads to significant loss of accuracy that cannot be controlled.

In view of these difficulties, we develop and apply the VSIE method. Using the fast discrete Fourier transform and taking into account that the VSIE kernels depend on the difference of arguments, we consider fast algorithms of the matrix-vector multiplication. Next, using iteration techniques, we perform efficient numerical solution of the initial scattering problems by the VSIE method [9].

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Recall main parameters that govern efficiency of a numerical algorithm: number T of arithmetic operations which is necessary for obtaining the sought-for solution with the required accuracy and volume M of the memory required for implementation of the algorithm.

Matrix–vector multiplication is the most laborious operation when iteration techniques are applied. Therefore, the number of these multiplications in the process of implementation of the algorithm will be called *the number of iterations*. The value of T may be estimated from the formula

$$T \approx L(T_A + T_0) + T_M.$$

Here L is the number of iterations, T_A is the number of arithmetic operations for multiplying a matrix by a vector, T_0 is the number of arithmetic operations for other calculations performed within one iteration, and T_M is the number of arithmetic operations for forming the SLAE matrix when an integral equation is discretized. As a rule, $T_0 << T_A$ и $T_M << LT_A$. The memory volume required for implementation of the algorithm can be estimated by the expression

$$M \approx M_A + M_{ITER}$$

Here M_A and M_{ITER} are the memory volumes required, respectively, for storing the SLAE matrix or the corresponding array and implementing the iteration procedure.

Quantities T_A, T_M, and M_A depend solely on the method of dicretization of the integral equation, while L, T_0, and M_{ITER} are governed by the iteration algorithm in use.

Since the VSIE kernels depend on the difference of arguments, we can provide the fulfillment of the condition $M_A \sim N$, where N is the matrix size using efficient dicretization.

The value of T_A is another important characteristic of the algorithm. If no special algorithms are used, then $T_A \approx N^2$, which causes abnormal computational expenses due to huge matrix dimensions. Using the fast discrete Fourier transform, it is possible to create an efficient algorithm with $T_A \sim NLOG(N)$, where $LOG(N)$ is a function taking values on the set of whole numbers and equal to the sum of all prime factors of number N. Obviously, this is a slowly varying function with respect to increasing; for example, $LOG(10^6) = 42$ and $LOG(10^8) = 56$.

Now consider minimization of the number of iterations L and quantities T_0 and M_{ITER} which are governed by the iteration algorithm. Usually, the integral equations under study are solved by the iteration algorithm called *Generalized Minimal Residual Algorithm* (GMRES) and its various modifications [4]. When this method is applied, the iterations parameters are determined in the course of calculations and depend on the current iteration index. Such methods may be called nonstationary iteration algorithms. GMRES-type algorithms are very popular; however, their implementation requires comparatively large amount of computer memory with $M_{ITER} \approx nN$, where n is the dimension of the Krylov subspace (note that the rate of convergence of iterations to the solution increases together with n). Also, the number of iterations required for providing reasonable accuracy of solution is often very large.

There is another family of iteration methods called stationary iteration algorithms for which the iterations parameters are determined before the iteration procedure. For a stationary iteration algorithm, the values of T_0 и M_{ITER} turn out to be minimal. These methods are usually applied for solving equations with selfadjoint operators for which the boundaries of the spectrum location on the real axis of the complex plane are known. The Chebyshev iteration and the method of simple iteration [6] belong to this family. For this methods we have $M_{ITER} \approx N$.

In Section 2 we describe the generalized method of simple iteration (GSI) for solving equations with nonselfadjoint operators [9] and construct a finite algorithm of finding an optimal iteration parameter in the case of arbitrary location of the operator spectrum on the complex plane. In Section 3 we describe the generalized Chebyshev iteration (GCI) applied for solving equations with nonselfadjoint operators. These techniques demand the knowledge of the spectrum localization on the complex plane. It is not possible to obtain this information for the majority of problems. However, one can do it in a number of important particular cases. In Section 4 we formulate the problems of electromagnetic scattering and reduce them to VSIEs. In Section 5 we generalize the results obtained in [1] and describe the domain of the spectrum location explicitly for low-frequency scattering problems and in the general case. In Section 6 we illustrate theoretical findings of this study by the results of computations obtained for a representative scattering problem, namely, low-frequency scattering of a plane electromagnetic wave from an inhomogeneous dielectric ball, and compare our iterative strategies and methods employing optimal parameters with some typical results obtained by a state-of-the-art algorithm. In Section 7 we discuss the obtained results and give recommendations concerning their applications.

2. Generalized simple iterations

Consider a linear operator equation

$$\hat{A}u = f, \tag{2.1}$$

in the Banach space E, where $\hat{A}$ is a bounded and therefore continuous operator (generally, nonselfadjoint).

Write Eq. (2.1) in the equivalent form

$$u - \hat{B}_\mu u = f/\mu. \tag{2.2}$$