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Richardson extrapolation-based sensitivity analysis in the multi-objective optimization of analog circuits

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ABSTRACT

The feasible solutions provided by a multi-objective evolutionary algorithm (MOEA) in the optimal sizing of analog integrated circuits (ICs) can be very sensitive to process variations. Therefore, to select the optimal sizes of metal–oxide–semiconductor field-effect-transistors (MOSFETs) but with low sensitivities, we propose to perform multi-parameter sensitivity analysis. However, since MOEAs generate feasible solutions without an explicit equation, then we show the application of Richardson extrapolation to approximate the partial derivatives associated to the sensitivities of the performances of an amplifier with respect to the sizes of every MOSFET. The proposed multi-parameter sensitivity analysis is verified through the optimization of a recycled folded cascode (RFC) operational transconductance amplifier (OTA). We show the behavior of the multi-parameter sensitivity approach versus generations. The final results show that the optimal sizes, selected after executing the sensitivity approach, guarantee the lowest sensitivities values while improving the performances of the RFC OTA.

1. Introduction

To have a general idea on analog integrated circuit (IC) sizing strategies developed by researchers and companies during the last 20 years, an overview on the classification and a brief description of the majority of them can be found in [1]. Although these works and other recently published strategies [2–4] provide good sizing solutions, still the analog design community deals with the hard open problem related to process variations [5,6]. In this manner, we propose to perform multi-parameter sensitivity analysis to the feasible solutions provided by a multi-objective evolutionary algorithm (MOEA), with the goal to select the optimal sizes of an analog IC but with low sensitivities. Because very often, the best feasible solutions meeting extreme performance requirements are located at some peripherals of the feasible solution space, but some variability in the design parameters can transform a best solution to a worst one [7,6,8,5].

Since our proposed multi-parameter sensitivity analysis is performed from numerical data instead of using explicit equations, we propose to apply numerical finite differences and Richardson extrapolation [9–12], to approximate the partial derivatives associated to the sensitivities of the sizing relationships W/L (width/large) of the MOSFETs. These processes are performed in two domains: variables W/L (design parameters) and objectives, where both are evaluated by linking HSPICE®.

The first step of our proposed approach consists on conventional optimization by applying the MOEA called non-dominated sorting genetic algorithm (NSGA-II) [13]. The second step is devoted to perform multi-parameter sensitivity analysis

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for all feasible solutions in the Pareto front. The goal is to discriminate those feasible solutions located in a delicate point that does not support the natural variations of the fabrication processes, i.e. those having large sensitivities.

2. Multi-objective optimization

The optimization stage is performed by applying the MOEA NSGA-II, to minimize a problem of the form [14]:

$$\begin{aligned} &\text{minimize } \mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x})]^T \\ &\text{subject to } h_k(\mathbf{x}) \geqslant 0, \quad k = 1 \ldots p, \end{aligned} \tag{1}$$

where function $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^m$, $\mathbf{x} = [x_1, \ldots, x_n]^T$ is the decision vector and n is the number of variables; $\mathbf{x} \in X$, where $X \subset \mathbb{R}^n$ is the decision space for the variables. Every objective function $f_j(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R},\ j = 1 \ldots m\ (m \geqslant 2)$ and $h_k(\mathbf{x}),\ k = 1 \ldots p$ are performance constraints.

Very often, since the objectives in (1) contradict each other, no point $\mathbf{x} \in X$ minimizes all the objectives simultaneously. The best tradeoffs among the objectives can be defined in terms of Pareto optimality [15–17].

The NSGA-II Algorithm is based on Pareto ranking. First, two populations (P_o and Q_o) are generated, each one of size N. The NSGA-II procedure in each generation consists of rebuilding the current (t) population (R_t) from the two original populations (P_t and Q_t). Next, through a nondominated sorting procedure all solutions in R_t are ranked and classified in a family of sub-fronts [13]. In the next step, a new offspring (P_{t+1}) is created from the current population R_t (previously ranked and ordered by sub-front number), with the goal to choose from a population of size $2N, N$ solutions belonging to the first sub-fronts. Besides, the last sub-front could be greater than necessary, and then a measure ($i_{distance}$) is used to preserve diversity by selecting the solutions that are far from the rest [18]. To build new generations we use differential evolution (DE) [19], as genetic operator.

Regarding to circuit sizing, each variable x represents the width (W) or length (L) of the MOSFETs. Usually, those values are integer-multiples of the minimum value allowed by the fabrication processes. In this manner, if the W/L relationship is expressed in multiples of the minimum L, then the DE operator is performed by rounding W/L to the closer multiple of the minimum L.

3. Multi-parameter sensitivity analysis

The relative or normalized sensitivity (S) can be defined as the cause and effect relationship between the circuit elements variations, and the resulting changes in the performances response [20,21]. Furthermore, in the design of analog ICs the lowest sensitivity is very desired.

Let $f_i(\mathbf{x})$ be an objective function (performance response), where $\mathbf{x} = [x_1, \ldots, x_n]^T$ are the design variables. It is possible to relate small changes in the response of the performance (∂f_i, $i \in [1, m]$) to variations in the design variables ($\partial x_j,\ j \in [1, n]$). It leads us to the single parameter sensitivity definition given by,

$$S_{x_j}^{f_i} \simeq \frac{x_j}{f_i} \frac{\partial f_i}{\partial x_j}. \tag{2}$$

According to (2), there is one sensitivity for each objective function in $\mathbf{f}$ (see (1)) and for each variable in $\mathbf{x}$. Then, it is possible to define the multi-parameter sensitivity which sums the different single sensitivities regarding the different variables for each objective as follows [21]:

$$S^{f_j} = \sqrt{\sum_{i=1}^{n} \left| S_{x_i}^{f_j} \right|^2 \cdot \sigma_{x_i}^2}, \tag{3}$$

where $S_{x_i}^{f_j}$ is calculated by (2), σ_{x_i} is a variability parameter of x_i and the square root is used to preserve the same units.

The performances of an analog IC are evaluated using HSPICE®, and they are considered as the objective functions. As one can infer, using a numerical circuit simulator, there is not possibility to derive an explicit equation for each performance or objective function. Therefore, in order to calculate the partial derivative required by (2), the Richardson extrapolation described by (4), is used herein:

$$\frac{\partial f_i}{\partial x_j} \approx \frac{g_i(\mathbf{x}, j, d) - g_i(\mathbf{x}, j, -d)}{2d}, \quad \text{with } d \to 0, \tag{4}$$

where function g_i is defined as:

$$\begin{aligned} &g_i : \mathbb{R}^n \to \mathbb{R}, \\ &g_i(\mathbf{x}, j, d) = f_i(\mathbf{y}) \,|y_k = x_k \text{ for } k \neq j \text{ and } y_j = x_j + d. \end{aligned} \tag{5}$$

In (4), d is a step parameter that is updated in each iteration [22], for this case $d = 2^{-u} d_{u-1}, d_0$ is assigned to an initial value and u is the current iteration. The recursive calculation continues until a tolerance error, as stopping criterion (δ), is reached.