Applied Mathematics and Computation 222 (2013) 381–390

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Stochastic adaptive synchronization for time-varying complex delayed dynamical networks with heterogeneous nodes ☆

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ARTICLE INFO

Keywords:
Stochastic dynamical network
Synchronization
Time-varying coupling strength
Adaptive control

ABSTRACT

Stochastic synchronization for a time-varying complex delayed dynamical networks with heterogeneous nodes is studied in this paper, using adaptive synchronization scheme for detailed analysis and discussion. Based on Lyapunov stability theory, the properties of Weiner process (Brownian motions), some controllers and adaptive laws are designed to ensure achieving stochastic synchronization of the complex delayed dynamical networks model. A sufficient condition is given to ensure that the proposed controlled networks model is globally asymptotically synchronized in mean square sense. A simulation example is presented to show the effectiveness and applicability of the proposed results.

1. Introduction

Complex networks [1–6] exist in all fields of sciences, nature and human societies. Over the past decade, complex networks have been studied intensively in various disciplines, such as computer networks, the World Wide Web, food webs, neural networks, electrical power grids and citation networks of scientists, among many others, even reputation computation in virtual organizations [7]. The investigation of complex dynamical networks is of great importance, and many systems in science and technology can be modelled as complex networks. For coupled complex networks, one prominent phenomenon is the synchrony of all dynamical nodes, which can well demonstrate many natural phenomena. Therefore, synchronization of complex networks of dynamical nodes has received a great deal of attention [8–12]. By using the Lyapunov functional method, or taking the synchronization manifold and linear matrix inequality approach, several sufficient conditions have been derived for ensuring the synchronization of various complex networks [8,9]. Some controllers have been commonly used, such as feedback and delayed feedback controllers [13,14], nonlinear adaptive feedback controllers [15–21], and so forth.

Recently, stochastic systems have received increasing attention and stochastic modeling has played an important role in many scientific and engineering applications. In fact, signals transmitted between nodes of complex networks are unavoidably subject to stochastic perturbations from environment, which may cause information contained in these signals to be lost [22]. Therefore, transmitted signals may not be fully detected and received by other subsystems. This can have a great influence on the behavior of complex networks. Therefore, to investigate and simulate more realistic complex networks, the noise effect should be taken into account in modeling the networks. Many fundamental results about deterministic system

☆ This work was supported by the National Natural Science Foundation of China under Grant No. 60974139, the Fundamental Research Funds for the Central Universities of China under Grant No. 72103676, the Scientific Research Fund of Yunnan Education Department under Grant No. 2012Z150C, and the Research Subject of Lincang Teacher's College under Grant No. LCSZL2012005.

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http://dx.doi.org/10.1016/j.amc.2013.07.030

have been extended to stochastic system. Stability of stochastic delayed systems has been a focal topic for research due to the uncertainties existing in the real-world systems [14,23,24]. In particular, synchronization in coupled stochastic networks has also been carefully studied [13,25,26]. The authors of [22] introduced the stochastic coupling term to model of linearly coupled neural networks and studied its complete synchronization. Authors of [27] studied the exponential stochastic synchronization problem for coupled neural networks with stochastic noise perturbations. In [28], adaptive synchronization of coupled stochastic networks subject to random perturbations is studied, with a referenced node introduced as the target node for synchronization. Authors of [29] studied the finite-time stochastic synchronization problem for complex networks with stochastic noise perturbations. Authors of [30] studied stochastic synchronization for a new array of coupled discrete time stochastic complex networks with randomly occurred nonlinearities and time delays. Although these studies reflect the complexity of the network structure, they focused on identical nodes. However, in many real world systems, such as laser arrays, finance and trade systems, manufacture network systems and biological systems, it is hardly the case that every component can be assumed to be identical. As a result, more and more applications of chaos synchronization in secure communications make it much more important to synchronize two different chaotic systems in recent years [31]. In the real world, most nodes of the complex dynamical networks are different. The authors of [32,33] studied impulse control and pinning control of complex networks, respectively. The authors of [34] introduced a dynamical network model with different nodes, and investigated its pinning control onto a homogeneous stationary state. In Ref. [35], synchronization criterion of a complex dynamical network with coupling time-varying delays via sampled-data control was presented. The synchronization of complex dynamical networks with similar nodes and coupling time-delay was proposed in Ref. [36]. In Ref. [37], cluster synchronization in a network of non-identical dynamic systems was studied, several sufficient conditions are obtained by using the Schur unitary triangularization theorem. Zhao et al. [38] presented a framework for global synchronization of dynamical networks with nonidentical nodes, and gave several criteria for synchronization using free matrices for both cases of synchronizing to a common equilibrium solution of all isolated nodes and synchronizing to the average state trajectory. However, to the best of our knowledge, up to now little work is reported the stochastic synchronization of nonidentical complex dynamical networks with time delays and unknown time-varying coupling strength.

Motivated by the above discussions, in this paper, we introduce a new model of complex delayed dynamical networks, which includes not only the time-varying coupling strength, but also unknown time-varying diffusive coupling delay. As well known, adaptive control can effectively deal with constant parametric uncertainties for systems. It still is an open problem how to control a system with unknown time-varying parameters. By virtue of properties of Weiner process and inequality techniques, suitable adaptive controllers are designed to ensure stochastic synchronization for the complex delayed dynamical networks with stochastic perturbations. Numerical analysis and its simulations demonstrate the effectiveness of our new results.

The rest of this paper is organized as follows. In Section 2, a new model of complex delayed dynamical networks with time-varying coupling strength is presented, some necessary assumptions, definitions, and lemmas are also given in this section. Our main results and their rigorous proofs are described in Section 3. In Section 4, an illustrative example and its simulations are provided to demonstrate the effectiveness of our results. Finally, in Section 5, conclusion and future investigation directions are given.

2. Problem formulation

Now suppose that, at some time, the network consists of N differently and diffusively coupled nodes, with each node being an n-dimensional dynamical system. For simplicity, we consider two kinds of nodes and assume the first $l(1 < l < N)$ nodes are the same. The state equations of the network are

$$\begin{cases} dx_i(t) = \left[f(x_i(t)) + \sum_{j=1}^{N}\varphi(t)a_{ij}\boldsymbol{\Gamma}x_j(t-h_1(t))\right]dt + g_i(x_1,\ldots,x_N)d\omega_i(t), & i=1,\ldots,l, \\ dx_i(t) = \left[g(x_i(t)) + \sum_{j=1}^{N}\varphi(t)a_{ij}\boldsymbol{\Gamma}x_j(t-h_2(t))\right]dt + g_i(x_1,\ldots,x_N)d\omega_i(t), & i=l+1,\ldots,N, \end{cases} \tag{1}$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t)]^T \in \mathbf{R^n}$ is the state vector representing the state variables of node i,$f, g : \mathbf{R^n} \times \mathbf{R} \to \mathbf{R^n}$ are smooth nonlinear vector-valued function, $h_1(t), h_2(t)$ are the time-varying delay and $h_1 : \mathbf{R}_+ \to [0, \hbar_1], h_2 : \mathbf{R}_+ \to [0, \hbar_2]$ are Borel measurable functions, $\varphi(t)$ represents the unknown time-varying coupling strength, $a_{ij} = a_{ji}$ are the coupling element, if there exists a connection between node i and node $j(j \neq i)$, then $a_{ij} = a_{ji} > 0$, else $a_{ij} = a_{ji} = 0$. The inner coupling matrixes $\boldsymbol{\Gamma} = diag(\gamma_1, \ldots, \gamma_n)$ is non-negative definite. The constant matrix $\mathbf{A} = (a_{ij})_{N\times N}$ describe the linear coupling configuration of the network, which satisfies

$$a_{ij} \geq 0,\ for\ i \neq j,\ and\ a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij}, i = 1,2,\ldots,N$$

and $g_i \in C(\mathbf{R^n} \times \ldots \times \mathbf{R^n}, \mathbf{R^{n\times n}})$ is the noise intensity function matrix, $\omega_i(t) = (\omega_{i1}(t), \omega_{i2}(t), \ldots, \omega_{in}(t))^T$ is an n-dimensional Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$ generated by