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Optimal portfolio and consumption with habit formation in a jump diffusion market ☆

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ABSTRACT

This paper studies optimal portfolio and consumption selection with habit formation in a jump diffusions incomplete market in continuous-time. The stochastic maximum principle for jump processes is applied to solve habit-forming utility maximization problem. We transform this problem into the case not involving habit formation in mechanically. Then the solution in the state feedback form is given. The relationship between maximum principle and dynamic programming is employed to get the expression of the relative risk aversion (RRA) coefficient and its distribution. Finally, for a special case, the stationary mean of the RRA coefficient is obtained and the numerical experiment indicates our model with jump diffusions is better than the model in [1] to resolve the equity premium puzzle in a way.

1. Introduction

Optimal portfolio and consumption selection are the predominant investment problems in finance. Following two seminal works by Samuelson [2] and Merton [3], a large number of papers have examined the optimal consumption and consumption selection. To describe investors preference over consumption, the traditional models typically use the Constant Relative Risk Aversion (CRRA) utility function, which assumes that investors judge their well-being by reference to the absolute level of their consumption, see Duffie [4], Karetzas and Shreve [5].

However, Mehra and Prescott [6]suggested that the traditional models could not explain the equity premium puzzle. They estimated the annual real return on the S&P500 index in the 1889–1978 period to have mean 6.98%, however, the mean annual real rate of return on 90-day Treasury bills to be 0.8%, in the 1931–1978 period and the equity premium to be 6.18 %. The covariance of the annual real return on the S&P500 index and the consumption growth rate was 0.00375. According to Comsumption Capital Asset Pricing Model (CCAPM), the RRA coefficient, which gets from the equity premium being divided by the covariance, equals about 16. However, 16 is far greater than $2 \sim 3$ which is estimated by questionnaires according to experimental economics. This is why we call it as the equity premium puzzle.

Constantinides [1] and Abel [7]successfully explained the equity premium puzzle via habit formation. They claimed that the utility of a given current consumption stream was decreasing in the habit level determined by the past consumption, and the presence of habit formation may give rise to major changes in the consumption and saving decisions of the representative agent. Essentially habit persistence drives a wedge between the relative risk aversion of the representative agent and the intertemporal elasticity of substitution in consumption.

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Habit Formation preference has become a popular alternative tool of consumption optimization problems. A large body of papers study optimal consumption-portfolio policies with habit formation in a complete market. Detemple and Zapatero [8,9] proved the existence of optimal consumption-portfolio policies for several utility functions. In the same framework, Schroder and Skiadas [10]presented an isomorphism between optimal portfolio selection with utilities incorporating linear habit formation, and corresponding models without habit formation. The isomorphism can be employed to mechanically transform known solutions not involving habit formation to corresponding solutions with habit formation. Nakagawa [11] used same method to study asset pricing with stochastic habit formation. To list a very small subset of the existing literature in optimal investment and consumption problems with habit formation, we refer to Englezos and Karatzas [12], Santos and Veronesi [13], Choi [14], Lichtendahl et al. [15].

However, in the realistic world, the financial market is incomplete. For example, the number of risk in the market is more than risk asserts. Furthermore, there are accidents which are similar to jumps in the realistic world, like the financial crisis and catastrophes. Therefore, jump diffusions process is a popular alternative tool to model the jump risks. Stochastic processes with jump diffusions have become more and more popular for modeling fluctuations in financial mathematics, both for option pricing and portfolio selection, see Framstad et al. [16,17], Kou [18], Guo and Xu [19], Nunno et al. [20], Wang and Wu [21], Shi and Wu [22], Wang et al. [23].

There is little peer-reviewed literature about optimal investment and consumption problems with habit formation in incomplete markets. Yu [24] studied utility maximization with habit formation in incomplete semimartingale markets. Muraviev [25] provided a detailed characterization of the optimal consumption stream for the additive habit-forming utility maximization problem, in a framework of general discrete-time incomplete markets and random endowments.

Unfortunately, research on optimal portfolio and consumption selection with habit formation in jump diffusions incomplete markets in continuous-time is literally nil according to our best knowledge. In this literature, we employ stochastic processes with jump diffusions to model an incomplete market where are at least two risks including the Brownian motion and Lévy process and only one risk assert. Then we use stochastic maximum principle for the optimal control of jump diffusions to solve habit-forming utility maximization problem of optimal portfolio and consumption choice.

The recent literature in stochastic control of jump diffusions focued on sufficient stochastic maximum principle and corresponding dynamic programming, see Framstad et al. [17], Øksendal and Sulem [26], Baghery and Øksendal [27], Shi and Wu [22]. In this paper, we develop the utility in [17] into habit-forming one, and employ transformation in [8,9,11] to get a utility without habit formation. By the same method in [17], we obtain solutions of optimal portfolio and consumption in the state feedback form.

From optimal solutions, we compute the expression of the RRA coefficient and study its distribution. For a special case, the stationary mean of the RRA coefficient is obtained. Another contribution of this literature is numerical experiment in this special which suggests our model with jump diffusions is better than the model in [1] to resolve the equity premium puzzle in a way.

The rest of this paper is organized as follows. In Section 2, we present the model for the underlying market. The solution of habit-forming utility maximization problem is studied in Section 3. In Section 4, we investigate the RRA coefficient. A special case and the numerical experiments are shown in Section 5 and conclusions are given in Section 6.

2. Problem formulation

Suppose we have a financial market with the following two investment possibilities:

(i) A risk free asset , whose price at time is given by

$$dS_0(t) = r_t S_0(t)dt,\ S_0(0) > 0, \tag{1}$$

(ii) A risky asset , whose price at time is given by

$$dS_1(t) = S_1(t^-)[\mu_t dt + \sigma_t dB(t) + \int_{R_0} \theta(t,z)\widetilde{N}(dt,dz)],\ S_1(0) > 0. \tag{2}$$

where $r_t \neq 0,\ \mu_t \neq 0,\ \sigma_t \neq 0$, and $\theta(t,z)$ are locally bounded deterministic functions, $t \in [0,T],\ z \in R_0 \subset \mathcal{R}/\{0\}$ and $T > 0; B(t)$ is a 1-dimensional Brownian motion; $\widetilde{N}(dt,dz) = N(dt,dz) - \nu(dz)dt$ is the compensated jump measure of $\eta(\cdot)$ on space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geqslant 0}), N(dt,dz)$ is the jump measure, and $\nu(dz)$ is the Lévy measure of Lévy process $\eta(\cdot)$, which is given by $\eta(\cdot) = \int_0^t \int_{R_0} \widetilde{N}(dt,dz),\ t \geqslant 0$. From the above settings, the risk is at least two dimensional including the Brownian motion and jump process but only one risky asset is in the market. Then the market in our model is incomplete.

We also assume that $\theta(t,z) \geqslant -1$, and

$$E\left[\int_0^t \int_{R_0} |1 + \theta(t,z)|^2 \nu(dz)dt\right] < \infty.$$