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Internal null stabilization for some diffusive models of population dynamics

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ABSTRACT

We investigate the large-time behavior of the solutions to some Fisher-type models with nonlocal terms describing the dynamics of biological populations with diffusion, logistic term and migration. Two types of logistic terms are taken into account. A necessary condition and a sufficient condition for the internal null stabilizability of the solution to a Fisher model with nonlocal term are provided. In case of null stabilizability (with state constraints) a feedback stabilizing control of harvesting type is proposed. The rate of stabilization corresponding to the feedback stabilizing control is dictated by the principal eigenvalue to a certain linear but not selfadjoint operator. A large principal eigenvalue leads to a fast stabilization to zero.

Another goal is to approximate this principal eigenvalue using a method suggested by the theoretical result concerning the large time behavior of the solution to a certain Fisher model with a special logistic term. An iterative method to improve the position (by translations) of the support of the feedback stabilizing control in order to get a larger principal eigenvalue, and consequently a faster stabilization to zero is derived. Numerical tests illustrating the effectiveness of the theoretical results are given.

1. Setting of the problems

After the pioneering work of Fisher [18] the mathematical modeling of spatially structured populations has been carefully analyzed, given rise to a flourishing literature on the diffusive biological models (see [14,27,28]). The local/nonlocal intra or interspecific interactions of one or several interacting populations species were taken into account by several authors (see e.g. [7,15,19,20]). The following Fisher-type model describes the dynamics of a single biological population species which is free to move in an isolated habitat Ω:

$$\partial_t y(x,t) - d\Delta y(x,t) = a(x)y(x,t) - k(x)y(x,t)^2, \quad x \in \Omega,\ t > 0.$$

Here $\Omega \subset \mathbf{R}^N$ ($N \in \{2,3\}$) is a bounded domain, with a smooth enough boundary $\partial\Omega, y(x,t)$ is the population density at position x and moment $t, d > 0$ is the diffusion coefficient, $a(x)$ is the natural growth rate at position x, and $k(x)y^2$ is a logistic term.

The logistic term describes a local intraspecific competition for resources. If a migration phenomenon occurs then the population dynamics is described by

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$$\partial_t y - d\Delta y = a(x)y - b(x)y + \int_\Omega l(x,x')b(x')y(x',t)dx' - k(x)y^2, \quad x \in \Omega,\ t > 0,$$

where $b(x) \geqslant 0$ denotes the rate of population that leaves position x and migrates to other locations in Ω, and $l(x,x') \geqslant 0$ represents the rate of population that leaves position x' and migrates to position x. Since the migration does not affect the size of the total population, the following condition holds

$$\int_\Omega l(x,x')dx = 1 \quad \text{a.e. } x' \in \Omega.$$

If we denote by $c(x) = a(x) - b(x)$ and $F(x,x') = l(x,x')b(x')$, and take into account that the habitat is isolated, then the population dynamics with diffusion, logistic term and migration may be described by the next system:

$$\begin{cases} \partial_t y = d\Delta y + c(x)y - k(x)y^2 + \int_\Omega F(x,x')y(x',t)dx', & (x,t) \in Q, \\ \partial_\nu y(x,t) = 0, & (x,t) \in \Sigma, \\ y(x,0) = y_0(x), & x \in \Omega, \end{cases} \tag{1.1}$$

where $Q = \Omega \times (0,+\infty), \Sigma = \partial\Omega \times (0,+\infty)$, and y_0 is the initial density of the population. $\int_\Omega F(x,x')y(x',t)dx'$ is a nonlocal term.

Since any biological population living in a habitat interact with other population species, it is of great importance to be able to precise if the population will persist or will extinct. Sometimes it is imperative for the ecosystem to introduce a control in order to eradicate the population or to diminish it (with persistence) to a level which do not endanger other population species in the habitat.

We investigate for the beginning the large-time behavior of the solution to (1.1) and of the solution to a model where the logistic term has a different form (which models a nonlocal intraspecific competition for resources):

$$\begin{cases} \partial_t y = d\Delta y + c(x)y - k_1 y(x,t) \int_\Omega y(x,t)dx \\ \qquad + \int_\Omega F(x,x')y(x',t)dx', & (x,t) \in Q \\ \partial_\nu y(x,t) = 0, & (x,t) \in \Sigma \\ y(x,0) = y_0(x), & x \in \Omega. \end{cases} \tag{1.1'}$$

For other studies concerning qualitative analysis of some reaction–diffusion systems with nonlocal terms we refer to some recent papers [10,11,16,19,20], whilst for an age-dependent population dynamics with diffusion (but without nonlocal term) we refer to [24].

Another problem to be investigated is the internal null stabilization for the following controlled model:

$$\begin{cases} \partial_t y = d\Delta y + c(x)y - k(x)y^2 + \int_\Omega F(x,x')y(x',t)dx' + \chi_\omega(x)u, & (x,t) \in Q \\ \partial_\nu y(x,t) = 0, & (x,t) \in \Sigma \\ y(x,0) = y_0(x), & x \in \Omega, \end{cases} \tag{1.2}$$

where $\omega \subset \Omega$ is an open subset with a smooth enough boundary such that $\Omega \setminus \overline{\omega}$ is a domain, and χ_ω is the characteristic function of ω. Here u is a control and represents a harvesting or infusion of population and acts only in ω.

The question to be investigated is the following one: "is it possible to find a control u so that the solution y^u of (1.2) to be nonnegative and to tend to 0 as $t \to +\infty$?"

Here are the hypotheses we are going to use throughout this paper:

(H1) $d,\ k_1$ are positive constants, $c \in L^\infty(\Omega)$;
(H2) $F \in L^\infty(\Omega \times \Omega), k,\ y_0 \in L^\infty(\Omega)$, and

$$0 \leqslant F(x,x') \quad \text{a.e. } (x,x') \in \Omega \times \Omega$$

$$k_0 \leqslant k(x), \quad 0 \leqslant y_0(x) \quad \text{a.e. } x \in \Omega,$$

where k_0 is a positive constant.

Definition 1.1. The population is internally null-stabilizable if for any $y_0 \in L^\infty(\Omega), y_0(x) \geqslant 0$ a.e. $x \in \Omega$ there exists a control $u \in L^\infty_{loc}(\overline{\omega} \times [0,+\infty))$ such that the solution y^u to (1.2) satisfies

$$0 \leqslant y^u(x,t) \quad \text{a.e. } x \in \Omega,\ \forall t \geqslant 0 \tag{1.3}$$

and

$$\lim_{t \to +\infty} y^u(t) = 0 \quad \text{in } L^\infty(\Omega). \tag{1.4}$$