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A delayed Lotka–Volterra model with birth pulse and impulsive effect at different moment on the prey ☆

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ABSTRACT

In this paper, we consider a Lotka–Volterra model with birth pulse and impulsive catching or poisoning for the prey at different moment and stage structure on the predator. We prove that all solutions of the system are uniformly ultimately bounded. Sufficient conditions of the global attractivity of predator-extinction periodic solution and the permanence of the system are obtained. These results show that impulsive effects on the prey play an important role for the permanence of the system. In this paper, the main feature is to introduce birth pulse and impulse catching into Lotka–Volterra model and to give pest management strategies.

1. Introduction

According to the reports of Food and Agriculture Organization of United Nations, the warfare between man and pests has lasted for thousands of years. With the development of society and the progress of science and technology, different methods are used in the process of pest management, for instance, chemical pesticides, biological pesticides, remote sensing and measure, computers, atomic energy etc. Some achievements have been obtained, but the warfare is not over and will continue. Of all methods, chemical pesticides seem to be a convenient and ecient one, because they can quickly kill a significant portion of a pest population.

The predator–prey models with stage structure for the predator were introduced and investigated by Hastings [1], Meng et al. [2], Zhang et al. [3] and Gourley and Kuang [4]. Since the immature predator takes τ units of time to mature and the death toll during the juvenile period should be considered, time delays have important biological meanings in age-structured models. Hence many stage structured models with time delay were extensively studied by Zhang et al. [3], Gourley and Kuang [4], Liu and Beretta [5] and Liu et al.[6]. In recently years, impulsive systems are found in many domains of applied sciences[7,8]. The investigation of impulsive delay differential equations is beginning, and impulsive delay differential equations have almost analyzed in theory by Liu and Ballinger [9]. Time delay and impulse are introduced into predator–prey models with stage structure, which greatly enriches biologic background. However, delay pest management models

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with impulsive effects at different moment have never been reported by now. In [10], Liu considered a delay differential equation:

$$\begin{cases} \left.\begin{array}{l} \dot{x}(t) = rx(t)\ln\frac{K}{x(t)} - \frac{\beta x(t)y_2(t)}{1+\alpha x(t)}, \\ \dot{y}_1(t) = \frac{\lambda\beta x(t)y_2(t)}{1+\alpha x(t)} - e^{-d\tau}\frac{\lambda\beta x(t-\tau)y_2(t-\tau)}{1+\alpha x(t-\tau)} - dy_1(t), \\ \dot{y}_2(t) = e^{-d\tau}\frac{\lambda\beta x(t-\tau)y_2(t-\tau)}{1+\alpha x(t-\tau)} - dy_2(t), \end{array}\right\} \quad t \neq nT, \\ \left.\begin{array}{l} x(t^+) = (1-\delta)x(t), \\ y_1(t^+) = y_1(t), \\ y_2(t^+) = y_2(t), \end{array}\right\} \quad t = nT,\ n = 1,2\ldots \\ (\varphi_1(s), \varphi_2(s), \varphi_3(s)) \in C_+ = C([-\tau_1, 0], R^3_+), \quad \varphi_i|(0) > 0,\ i = 1,2,3, \end{cases} \tag{1.1}$$

where $x(t),\ y_1(t),\ y_2(t)$ represent the densities of prey, immature and mature predator, respectively, r is the Gomportz intrinsic growth rate of the prey in the absence of the predator, the capacity rate K is concerned with the resources which maintain the evolution of the population, β is the predation rate of predator and λ represents the conversion rate at which ingested prey in excess of what is needed for maintenance is translated into predator population increase, d is the death rate of predator, δ $(0 \leqslant \delta < 1)$ represents partial impulsive harvest to preys by catching or pesticides, τ is the mean length of the juvenile period, α is the saturation which represents that a certain amount of predators can prey on a limited amount of preys, although the preys are numerous.

In [11], Clack has studied the optimal harvesting of the logistic equation, a logistic equation with out exploitation is as following:

$$\dot{x}(t) = rx(t)\left(1 - \frac{x(t)}{K}\right), \tag{1.2}$$

where $x(t)$ represents the density of the resource population at time t, r is the intrinsic growth rate, the positive constant K is referred as the environmental carrying capacity. Suppose that the population described by logistic Eq. (1.2) is subject to harvesting at rate $h(t) = Ex(t)$. Then the equation of the harvested population revise can be revised as following:

$$\dot{x}(t) = rx(t)\left(1 - \frac{x(t)}{K}\right) - Ex(t), \tag{1.3}$$

where E denotes the harvesting effort.

These models have invariably assumed that the mature population reproduce throughout the year, whereas it is often the case that births are seasonal or occur in regular pulses. The continuous reproduction of mature population is removed from the model, and replaced with an annual birth pulse. Combining (1.2) and (1.3), we can obtain a single population model with birth pulse and impulsive harvesting at different moments:

$$\begin{cases} \dot{x}(t) = -\mu x(t), & t \neq (n+l)T,\ t \neq (n+1)T, \\ \Delta x(t) = rx(t)\left(1 - \frac{x(t)}{K}\right), & t = (n+l)T, \\ \Delta x(t) = -px(t), & t = (n+1)T,\ n \in Z^+. \end{cases} \tag{1.4}$$

In this paper, we consider a Lotka–Volterra model with birth pulse and impulsive harvest for the prey, we show that there exists a globally attractive predator-extinction periodic solution when $R_1 < 1$. Furthermore, we also prove that the system is permanent if $R_2 > 1$.

2. Model formulation and preliminary information

There are many works concerning predator–prey system, and many good results are obtained [12–15]. The model we consider is based on the following predator–prey system

$$\begin{cases} x'(t) = x(t)(r - ax(t) - by(t)), \\ y'(t) = cx(t)y(t) - dy(t), \end{cases} \tag{2.1}$$

where $x(t)$ and $y(t)$ are densities of the prey and the predator, respectively, $r > 0$ is the intrinsic growth rate of prey, $a > 0$ is the coefficient of intraspecific competition, $b > 0$ is the per-capita rate of predation of the predator. d is the death rate of predator, c denotes the product of the per-capita rate of predation and the rate of conversing pest into predator. According to the nature of biological resource management, developing (2.1) by introducing birth pulse and impulsive harvesting on the prey at different moments. We consider the following impulsive delay differential equation: