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A mathematical model of a three species prey–predator system with impulsive control and Holling functional response ☆

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ABSTRACT

Taking into account periodic impulsive biological and chemical control for pest management at different fixed moment, a three species prey–predator system with Holling type II functional response was investigated. By using Floquet's theory and the small amplitude perturbation method, it was obtained that there exists an asymptotically stable preys-eradication periodic solution when the impulsive period is less than some critical minimum value (or the release amount of the predator is larger than some critical maximum value), and the system is permanent under the conditions that both the insecticidal effect and impulsive period are grater than some critical maximum values. Furthermore, it is obtained that IPM is more effective than any single one after comparison. Finally, numerical simulations are carried on to show the complex dynamic behavior of system.

1. Introduction

Integrated pest management (IPM) attempts to maximize the efficiency of control and at the same time minimize environmental damage by employing combinations of different methods of pest control which will complement each other and display synergism. During the recent years, the consequences of providing a predator–prey system with impulsive perturbations and its utility in pest control have been the topic of study of many scientists [1–4]. Impulsive differential equations are found in almost every domain of applied sciences [7,8] and have been studied in many investigations [9–11]. They generally describe phenomena which are subject to steep or instantaneous changes. Liu [1] developed the Holling type II predator–prey system with impulsive biological control given by:

$$\begin{cases} \left.\begin{aligned} x_1' &= x_1(t)[a_1 - x_1(t)] - \frac{x_1(t)x_2(t)}{1+ex_1(t)} \\ x_2' &= x_2(t)\left[\frac{m_1x_1(t)}{1+ex_1(t)} - d\right] \end{aligned}\right\} \quad t \neq n\tau, \\ x_2(t^+) = x_2(t) + p, \quad t = n\tau. \end{cases}$$

where $x_1(t),\ x_2(t)$ are the densities of the prey and predator at time t respectively. The predation term $x_1x_2/(1+ex_1)$, which is the functional response of the predator to change in the prey density, shows some saturation effect.

However, when examining more complicated cases where there are more food types available for predator, the above predation term does not reflect the consequences of adaptive behavior of individuals. Suppose that x_1 and x_2 are the densities

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of two prey species, while x_3 is the density of a predator species that preys upon x_1 and x_2. The predator consumes the prey x_i according to the functional response [5]:

$$\frac{\alpha_i x_i}{a_1 + b_1 x_1(t) + b_2 x_2(t)}, \quad i = 1, 2,$$

where α_i is the search rate of a predator for the prey x_i and $b_i = h_i \alpha_i$ where h_i is the expected handling time spent with the prey x_i to predator x_3. Based on these the predation terms, Sunita [6] investigated the three-species system with non-linear functional response:

$$\begin{cases} x_1'(t) = r_1 x_1(t)\left(1 - \frac{x_1(t)}{K_1}\right) - \frac{\alpha_1 x_1(t) x_3(t)}{a_1 + b_1 x_1(t) + b_2 x_2(t)}, \\ x_2'(t) = r_2 x_2(t)\left(1 - \frac{x_2(t)}{K_2}\right) - \frac{\alpha_2 x_2(t) x_3(t)}{a_1 + b_1 x_1(t) + b_2 x_2(t)}, \\ x_3'(t) = \left(-d + \frac{m_1 \alpha_1 x_1(t) + m_2 \alpha_2 x_2(t)}{a_1 + b_1 x_1(t) + b_2 x_2(t)}\right) x_3(t) \end{cases} \tag{1.1}$$

For system (1.1), the point $(0, 0, 0)$ is a equilibrium point but also a saddle point and $(0, 0, \alpha)$ with $\alpha > 0$ is not equilibrium point. So in this case the pests (preys) cannot be extinct, and the approach of this kind in pest control is not effective. Now, by introducing a constant periodic releasing natural enemies and spraying pesticide at different fixed moment. we will develop system (1.1) to the following impulsive differential equation:

$$\begin{cases} \left.\begin{aligned} x_1'(t) &= r_1 x_1(t)\left(1 - \tfrac{x_1(t)}{K_1}\right) - \tfrac{\alpha_1 x_1(t) x_3(t)}{a_1 + b_1 x_1(t) + b_2 x_2(t)}, \\ x_2'(t) &= r_2 x_2(t)\left(1 - \tfrac{x_2(t)}{K_2}\right) - \tfrac{\alpha_2 x_2(t) x_3(t)}{a_1 + b_1 x_1(t) + b_2 x_2(t)}, \\ x_3'(t) &= \left(-d + \tfrac{m_1 \alpha_1 x_1(t) + m_2 \alpha_2 x_2(t)}{a_1 + b_1 x_1(t) + b_2 x_2(t)}\right) x_3(t), \end{aligned}\right\} & \begin{aligned} &t \neq (k + l - 1)\tau, \\ &t \neq k\tau, k \in N, \end{aligned} \\ \left.\begin{aligned} x_1(t^+) &= (1 - \mu_1) x_1(t), \\ x_2(t^+) &= (1 - \mu_2) x_2(t), \\ x_3(t^+) &= x_3(t), \end{aligned}\right\} & t = (k + l - 1)\tau, k \in N, \\ \left.\begin{aligned} x_1(t^+) &= x_1(t), \\ x_2(t^+) &= x_2(t), \\ x_3(t^+) &= x_3(t) + p \end{aligned}\right\} & t = k\tau, k \in N, \end{cases} \tag{1.2}$$

where r_1, r_2 are the intrinsic growth rates and K_1, K_2 are the carrying capacities of the two preys. The parameter d denotes the death rate of predator and the constants m_1, m_2 are the rates of conversing preys into predator. The constants $\mu_i < 1 (i = 1, 2)$ are the death rate of the preys when chemical control is acted at time $t = (k + l - 1)\tau (k \in N)$ respectively and p is the release amount of the predator at time $k\tau$, where $0 \leqslant l \leqslant 1$. The parameter τ is the period of the impulsive effect and N is the set of all non-negative integers. All parameters are positive constants.

2. Preliminaries

First, we give some notations, definitions and lemmas which will be useful for our main results. Let $\mathbf{0} = (0, 0, 0)$, $R_+ = [0, +\infty)$, $R_+^3 = \{x \in R^3 | x \geqslant \mathbf{0}\}$. Denote $f = (f_1, f_2, f_3)$ the map defined by the right hand of system (1.2). Let $V : R_+ \times R_+^3 \to R_+$, then V is said to belong to class V_0 if:

(i) V is continuous in $(\tau_k, \tau_{k+1}] \times R_+^3$ and for each $x \in R_+^3$, $k \in N$, $\lim_{(t,y) \to (\tau_k^+, x)} V(t, y) = V(\tau_k^+, x)$ exists;
(ii) V is locally Lipschitzian in x.

Definition 2.1. Let $V \in V_0$ and $(t, x) \in (\tau_k, \tau_{k+1}] \times R_+^3$, the upper right derivative of $V(t, x)$ with respect to the impulsive differential system (1.2) is defined as

$$D^+ V(t, x) = \lim_{h \to 0^+} \sup \frac{1}{h} [V(t + h, x + hf(t, x)) - V(t, x)].$$

The solution of system (1.2) is denoted by $x(t) = (x_1(t), x_2(t), x_3(t)) : R_+ \to R_+ \times R_+^3$, and is continuously differential on $((k - 1)\tau, (k + l - 1)\tau]$ and $((k + l - 1)\tau, k\tau]$, $k \in N$ and $x((k - 1)\tau^+) = \lim_{t \to (k-1)\tau^+} x(t)$ and $x((k + l - 1)\tau^+) = \lim_{t \to (k+l-1)\tau^+} x(t)$ exist. The global existence and uniqueness of the solution of system (1.2) is guaranteed by the smoothness of f [7]. The following lemma is obvious.

Lemma 2.1. *Suppose $x(t)$ is a solution of (1.2) with $x(0^+) \geqslant \mathbf{0}$, then $x(t) \geqslant \mathbf{0}$ for all $t \geqslant 0$. And if $x(0^+) > \mathbf{0}$, then $x(t) > \mathbf{0}$ for all $t \geqslant 0$.*