



Free vibration analysis of multiple-stepped beams by the discrete singular convolution



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ARTICLE INFO

Keywords:

Discrete singular convolution
Multiple-stepped beam
Free vibration
Jump condition
Interpolation formulation

ABSTRACT

In this paper, free vibration of beams with multiple step changes are successfully analyzed by using the modified discrete singular convolution (DSC). The jump conditions at the steps are used to overcome the difficulty in using ordinary DSC for dealing with ill-posed problems. An n th-order interpolation polynomial is established on each side of the step. Then two or three jump conditions together with two continuous conditions at steps are used to relate the two polynomials. Five examples are analyzed. Results obtained by the modified DSC and existing solutions are compared. It is seen that the DSC results agree well with existing analytical, numerical, and experimental data. It is also found that the DSC results are even more accurate than the data obtained by the differential quadrature element method for much higher mode frequencies. The present research extends the application range of the DSC algorithm.

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1. Introduction

Multiple-stepped beams are widely used in many engineering fields. There are many examples of structures that can be modeled as beam-like elements, such as aircraft wing, long span bridges, and robot arms.

The free vibration analysis of multiple-stepped beams has been investigated by many researchers. Jang and Bert [1,2] derived the exact and numerical solutions to a single stepped beam. The transverse vibration of a beam with up to three step changes in cross-section with various combinations of boundary condition was analyzed by Naguleswaran [3]. It is seen that, however, obtaining the natural frequencies of beam structures with more steps is complicated and difficult if the analytical method is used. Thus various numerical methods, such as, the composite element method (CEM) [4], the adomian decomposition method (ADM) [5] and the differential quadrature element method (DQEM) [6], have been proposed to solve the problems. Lu and Huang [4] analyzed the vibration of multiple-stepped beams by using the CEM. The composite element method is basically a combination of the conventional finite element method (FEM) and the highly precise classical theory (CT). With enough degrees of freedoms (Dofs), CEM can yield accurate solutions. Mao [5] used the adomian decomposition method (ADM) to analyze the free vibration problem for multiple-stepped beam with arbitrary boundary conditions. To obtain the results, the stepped beam is divided into many sections, and the governing differential equation is established for each section. ADM seems not as efficient as other numerical method if tapered beams are involved. Recently, Wang [6] obtained highly accurate natural frequencies of multiple-stepped beams by using the DQEM. It is seen that the rate of convergence of DQEM is very high and the DQEM can yield very accurate results with a small number of grid points.

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The high-frequency vibrations are of crucial importance to aerospace structures such as aircraft, rotorcraft, satellite and space shuttle, jet fighter, rocket, and missile. Therefore, it is desirable and significant to develop an efficient method that can obtain both low and high order natural frequencies accurately. Despite much effort spent on the existing numerical methods in the past few decades, solving the high-frequency vibration problems numerically remains a challenging task.

The discrete singular convolution (DSC) algorithm, proposed by Wei [7] in 1999, is a method with such capability. The DSC possesses both the accuracy of the global method and the flexibility of the local method and can handle the challenging problem of vibrating at higher-order modes efficiently [8–10]. With much effort spent on the DSC algorithm, the method has been well developed now. The DSC can handle complex geometry and boundary conditions in many applications, such as, vibration analysis of plates with irregular internal supports [11], three-dimensional plate problems [12], free vibration of laminated composite thin plates [13] and shells [14], vibration of arbitrary straight-sided quadrilateral plates [15], vibration analysis of Mindlin plates with mixed edge supports [16], elastic wave propagations in one-dimensional structures [17], non-linear analysis of thin rectangular plates on Winkler–Pasternak elastic foundations [18], and non-linear buckling analysis [19]. Numerical results indicate that the DSC is a simple and reliable algorithm for analysis of well-posed mechanical problems. The DSC will suffer, however, difficulties in analyzing ill-posed mechanical problems, such as the title problem with discontinuous variations of geometry, if the jump conditions are not considered. To treat the geometric discontinuity, two methods proposed by Wei and his associates, the matched interface and boundary (MIB) method and interpolation formulations [20,21], are available. It is shown that these two methods are equivalent and have been successfully used in solving second order partial differential equations with geometric singularities. The problem of free vibration of multiple-stepped beams involves fourth order differential equations with discontinuities of the second- to fourth-order derivatives at the steps, however.

The objective of this paper is to utilize the DSC for the free vibration analysis of multiple-stepped beams. The interpolation formulation is to be employed to deal with the discontinuities of the second- to fourth-order derivatives at the steps. The ordinary DSC algorithm is briefly introduced first. The continuous and jump conditions at steps are then given and an n th-order polynomial on each side of the step is defined. Various free vibration problems of multiple-stepped beams are solved by using the modified DSC. Numerical results are compared to existing numerical solutions and data computed by using the commercial software (NASTRAN). This paper ends with a conclusion.

2. Theory

2.1. Brief introduction to the DSC

In the DSC algorithm, a function $w(x)$ and its n th-order derivative are usually approximated via a discretized convolution [22]

$$w^{(n)}(x) \approx \sum_{k=-M}^M \delta_{\sigma,\Delta}^{(n)}(x-x_k)w(x_k), \quad n = 0, 1, 2, \dots \quad (1)$$

where $2M+1$ is the computational bandwidth, $x_k (k = -M, -M+1, \dots, -1, 0, 1, \dots, M-1, M)$ are uniformly distributed grid points, and $\delta_{\sigma,\Delta}^{(n)}(x-x_k)$ is a collective symbol for the DSC kernels and given by

$$\delta_{\sigma,\Delta}^{(n)}(x-x_k) = \left(\frac{d}{dx}\right)^n \delta_{\sigma,\Delta}^{(0)}(x-x_k) \quad (2)$$

Although there are many DSC kernels available, the non-regularized Lagrange's delta sequence kernel (LK) is employed in the present study for simplicity, namely,

$$\delta_{M,k}(x-x_k) = \begin{cases} L_{M,k}(x-x_k) & \text{for } -\beta \leq x \leq \beta \\ 0 & \text{otherwise} \end{cases} \quad \text{for } M = 1, 2, \dots \quad (3)$$

where $\beta \leq L$ (L is the entire length of stepped beam), and $L_{M,k}(x)$ is the Lagrange interpolation defined by

$$L_{M,k} = \prod_{i=k-M, i \neq k}^{k+M} \frac{x-x_i}{x_k-x_i} \quad (M \geq 1) \quad (4)$$

2.2. Free vibration of stepped beams

Fig. 1 shows the sketch of a single stepped beam, where symbols L , a , E , I , A and ρ denote the beam length, location of the step, modulus of elasticity, moment of inertia, cross-sectional area, and the mass density of the element material, respectively. The governing differential equation of motion can be expressed as

$$\frac{d^2}{dx^2} \left(EI(x) \frac{d^2 w(x)}{dx^2} \right) = \rho A(x) \omega^2 w(x) \quad (5)$$

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