Applied Mathematics and Computation 219 (2013) 11258–11264

Contents lists available at SciVerse ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Soliton collision in a general coupled nonlinear Schrödinger system via symbolic computation

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ARTICLE INFO

Keywords:
General coupled nonlinear Schrödinger system
Soliton collision
Hirota method
Symbolic computation

ABSTRACT

A general coupled nonlinear Schrödinger system with the self-phase modulation, cross-phase modulation and four-wave mixing terms is investigated. The system is still integrable with the variable coefficients. Through the Hirota bilinear method, one- and two-soliton solutions are derived via symbolic computation. With the asymptotic analysis, it is found that the two-soliton solutions admit the inelastic and elastic collisions depending on the choice of solitonic parameters. A new inelastic collision phenomenon occurring in this system is that both the amplitudes of two components of each soliton get suppressed or enhanced after the collision, which might provide us with a different approach of signal amplification.

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1. Introduction

Able to stably transmit long distance in fibers with negligible attenuation, an optical soliton arises as a balance between the group velocity dispersion and self-phase modulation (SPM) [1–4]. Dynamic of optical-soliton propagation in a nolinear fiber is described by the nonlinear Schrödinger (NLS) equation [5]. In certain physical situations, solitons propagate simultaneously in multiple fields with different frequencies or polarizations, which can be modeled by the coupled NLS (CNLS) equations [6]. The CNLS equations have such applications as in the soliton wavelength division multiplexing [7], soliton switch in birefringent optical fibers [8,9], and multichannel bit parallel-wavelength optical fiber network [10].

Work has been done on the study of soliton propagation and collision in the CNLS systems [11–15]. For example, in Ref. [11], two-soliton solutions for the Manakov system (2-CNLS equations with special parametric choice) have been obtained and a shape-changing collision has been distinguished. Afterwards, the analysis has been extended to the case of N-CNLS system, and the similar property has been found that the energy can transfer from one soliton to another after collision [14,15].

In this paper, we will consider a general CNLS system, which is given as [16]

$$\begin{aligned} &ip_t + p_{xx} + 2(\alpha|p|^2 + \beta|q|^2 + \gamma pq^* + \gamma^* qp^*)p = 0, \\ &iq_t + q_{xx} + 2(\alpha|p|^2 + \beta|q|^2 + \gamma pq^* + \gamma^* qp^*)q = 0, \end{aligned} \tag{1}$$

where p and q are the complex amplitudes of the electrical fields in the two orthogonal polarizations, the subscripts t and x denote the temporal and spatial partial derivatives, and the terms with α and β respectively represent the SPM and cross-phase modulation (CPM) effects, and the last two terms including γ and γ^* describe the four-wave mixing effects, the asterisk

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http://dx.doi.org/10.1016/j.amc.2013.04.013

and subscripts denote the complex conjugation and partial derivatives, respectively. It is noted that α and β are real constants, and γ is a complex one.

When $\alpha = \beta$ and $\gamma = 0$, System (1) reduces to the Manakov system [17]. When $\alpha = -\beta$ and $\gamma = 0$, System (1) reduces to the $N = 2$ case considered in Ref. [14]. With the arbitrary coefficients $\alpha,\ \beta$ and γ, System (1) remains integrable and its Lax pair has been presented in Ref. [16], where the N-soliton solutions have been derived through the Riemann–Hilbert method, but the soliton collisions are only presented in the special case $\alpha = \beta = 0$ and $\gamma = 1$.

In this paper, we will concentrate on the general case $\alpha = \beta \neq 0$ and analyze the collision property with different choices of parameters. With symbolic computation [18–21], this paper will be arranged as follows. In Section 2, the bilinear form for System (1) will be derived, and the analytic one- and two-soliton solutions will be presented. In Section 3, through the asymptotic analysis on the two-soliton solutions, it will be found whether the collision between solitons is elastic or inelastic depends on the choice of soliton parameters. Further, owing to the existence of four-mixing wave, the inelastic collision shows some new feature, which can not be seen in the Manakov system but provide the possibility for future application to signal amplification. Section 4 will be our conclusions.

2. Bilinear form and soliton solutions

In order to understand the dynamics of System (1), it is essential to obtain the soliton solutions associated with the system. Via the Hirota bilinear method [22], one- and two-soliton solutions for System (1) can be deduced, and the procedure can be extended to obtain N-soliton solutions [23]. Nevertheless, we will concentrate on the study of one- and two-soliton solutions for System (1) in this paper.

System (1) can be expressed in the bilinear form

$$\left(iD_t + D_x^2\right)(g \cdot f) = 0, \tag{2a}$$

$$\left(iD_t + D_x^2\right)(h \cdot f) = 0, \tag{2b}$$

$$D_x^2(f \cdot f) = 2(\alpha|g|^2 + \beta|h|^2 + \gamma g h^* + \gamma^* h g^*), \tag{2c}$$

with the following transformations

$$p = \frac{g}{f}, \quad q = \frac{h}{f}, \tag{3}$$

where g and h are the complex functions of t and x, and f is a real one. D_x and D_t are the bilinear differential operators [22] defined by

$$D_x^m D_t^n (f \cdot g) = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^n f(x,t) g(x',t')|_{x'=x,\, t'=t}, \tag{4}$$

where m and n are the positive integers, x' and t' are the formal variables. Eq. (2) can be solved by introducing the following power series expansions as

$$g = \varepsilon g_1 + \varepsilon^3 g_3 + \varepsilon^5 g_5 + \cdots, \quad h = \varepsilon h_1 + \varepsilon^3 h_3 + \varepsilon^5 h_5 + \cdots, \quad f = 1 + \varepsilon^2 f_2 + \varepsilon^4 f_4 + \varepsilon^6 f_6 + \cdots,$$

where g_j's and h_j's $(j = 1, 3, 5, \ldots)$ are the complex functions of t and x, f_n's $(n = 2, 4, 6, \ldots)$ are the real ones which will be determined later, and ε is the formal parameter.

2.1. One-soliton solutions

In order to obtain one-soliton solutions for System (1), the power series expansions for g, h and f are terminated as

$$g = \varepsilon g_1, \quad h = \varepsilon h_1, \quad f = 1 + \varepsilon^2 f_2. \tag{5}$$

Substituting Eqs. (5) into Eq. (2) and collecting the terms with the same power of ε, we have the following solutions

$$g_1 = a_1 e^{\theta_1}, \quad h_1 = b_1 e^{\theta_1}, \quad f_2 = e^{\theta_1 + \theta_1^* + c_1}, \tag{6}$$

with

$$\theta_1 = k_1 x + i k_1^2 t, \quad e^{c_1} = \frac{|a_1|^2\alpha + |b_1|^2\beta + a_1 b_1^* \gamma + a_1^* b_1 \gamma^*}{(k_1 + k_1^*)^2},$$

where $a_1,\ b_1$ and k_1 are all complex constants. The resulting one-soliton solutions for System (1) are given as

$$p = \frac{a_1 e^{\theta_1}}{1 + e^{\theta_1 + \theta_1^* + c_1}} = \frac{a_1}{2} e^{-\frac{c_1}{2}} e^{i\theta_{1I}} \operatorname{sech}\left(\theta_{1R} + \frac{c_1}{2}\right), \tag{7}$$

$$q = \frac{b_1 e^{\theta_1}}{1 + e^{\theta_1 + \theta_1^* + c_1}} = \frac{b_1}{2} e^{-\frac{c_1}{2}} e^{i\theta_{1I}} \operatorname{sech}\left(\theta_{1R} + \frac{c_1}{2}\right), \tag{8}$$

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