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Open-loop optimal controller design using variational iteration method

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ABSTRACT

This article presents a design approach of a finite-time open-loop optimal controller using Pontryagin's minimum principle. The resulting equations constitute a two-point boundary-value problem, which is generally impossible to solve analytically and, furthermore the numerical solution is difficult to obtain due to the coupled nature of the solutions. In this paper, the variational iteration method is adopted to easily solve Hamilton equations by use of iteration formulas derived from the correction functionals corresponding to Hamilton equations. The proposed approach allows to derive the numerical solution of the optimal control problem but an analytical or approximate expression of the optimal control law can often be obtained as a function of the time variable, depending on the nature of the control problem, which is simple to implement. The different possible forms of control law that can be attained following the proposed design approach are illustrated by four application examples.

1. Introduction

Optimal control is the process of finding a control law that minimizes a performance index for a dynamic system over a period of time [1–3]. Optimal control has found applications in many different fields and represents one of the most challenging problem in the context of industrial, medical and economical applications. The enormous advances in computing power have enabled the application of optimal control methods to complex problems [3].

To solve an optimal control problem (OCP), several approaches are proposed in the literature [3–8]. However, two main approaches can be distinguished, direct and indirect methods [4]. Direct methods are based on collocation or a parametrization of control and possibly state variables [9], or iterative dynamic programming [4,6,10], and the OCP is solved by efficient nonlinear programming which presents good convergence properties and conveniently handles constraints. Thus, direct methods are purely numerical [3,4,11,6]. Indirect methods are based on the Variational Calculus [12] from which Hamilton–Jacobi theory and Pontryagin's principle of minimum or maximum [13,14] are issued for systems described by continuous state-space models and dynamic Programming based on Bellman's principle of optimality [15,1,16] for systems described by discrete state-space models. This results in a nonlinear two-point boundary-value problem which is difficult to solve. Thus, indirect methods possess an analytical basis, and in some rare cases, fairly simple optimal problems can be solved analytically, for instance when the performance index is quadratic and the dynamic system is linear [11,17]. Furthermore, the presence of active constraints on the inputs or on the states renders the problem more difficult to handle than in

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the case of direct methods [18]. Thus, in most cases, they need extensive numerical calculation together with advanced numerical methods.

To implement an optimal controller, two configurations are possible: open-loop optimal control (OLOC) and closed-loop optimal control (CLOC). Following [11], these configurations are equivalent, the difference being related to the form of the control law. In the case of OLOC, the control law is obtained as a time function whereas in the case of CLOC, the control law is obtained as a state feedback. The implementation of OLOC only requires a digital machine to perform the necessary numerical calculations, generally off-line. Note that CLOC is generally difficult to obtain except in the case of infinite-time optimal control problem of a linear system with quadratic performance index, known as the LQ problem [11]. In the case of finite-time LQ problem, a matrix differential Riccati equation, i.e a set of nonlinear differential equations, must be solved, and the solution is generally carried out numerically [19,20]. In addition, the practical implementation of CLOC requires the estimation of the entire state of the dynamic system like in LQG control, which makes observability of the system a primary question [21].

Among analytical methods developed for dynamic optimization problems, Pontryagin's minimum principle [14] is the most powerful and attractive approach. Nevertheless, the optimal control law is determined by solving a two-point boundary-value problem, given by a set of state and costate relations called Hamilton equations with appropriate boundary conditions, which is difficult to solve analytically or numerically using indirect methods because of opposite directions of integration for these coupled equations and possible constraints. The shooting method is commonly used to solve this two-point boundary-value problem [10,22]. Its principle consists in finding the unknown initial boundary conditions of costate variables so that Hamilton equations can be integrated together in the same direction. The main drawback of the shooting methods is that they are difficult to make converge because initial conditions for the costate variables should be guessed [10]. In addition, the numerical integration in the same direction of Hamilton equations is very often unstable [23]. To overcome these difficulties, direct methods are well adapted, even if the accuracy of indirect methods is claimed better.

The minimum principle remains the most used tool for solving both unconstrained and constrained optimal control problems. Several approaches have been proposed in the literature to deal with optimal control problem with constraints depending on their nature using the minimum principle. The main idea of the different proposed approaches consists in converting the original optimal control problem with various constraints [24] to a new one without constraints [18], then to apply the minimum principle. Equality constraints can be taken into account in the criterion to be minimized by use of Lagrange multipliers [25,26] and inequality constraints by use of Karush–Kuhn–Tucker-multipliers or use of slack variables [27,11]. Then the usual Hamilton–Jacobi method or Pontryagin's minimum principle can be used with an augmented number of variables. Graichen and Petit[18] transforms a constrained optimal control problem into an unconstrained one by a modification of the system dynamics under a normal form which incorporates the constraints. Graichen et al. [28] transforms an inequality-constrained optimal control problem into an equality-constrained one using saturation functions. Numerically, penalty functions [29,26,27,11] and interior-point methods [30] can also be used in the case of inequality constraints.

To solve ordinary differential equations, various approximate analytical methods have been proposed [31]. The commonly used methods are the Adomian Decomposition Method [32], the Homotopy Perturbation Method [33] and the Variational Iteration Method [34]. The comparison of the variational iteration method with the Adomian method [32] and the homotopy perturbation [33] reveals that the variational iteration method,which is nothing else but Picard–Lindelof iterative procedure [31,35], is more powerful and gives an approximation of higher accuracy and exact solutions if they exist.

Thus, in this paper, a new design approach of OLOC using Pontryagin's minimum principle is proposed. The main idea consists in using the variational iteration method [34,36] to get a solution, which can be analytical, approximate analytical or numerical depending on the nature of the nonlinearities of the optimal control problem. Thus, an approximate optimal control law, as a time function, can be easily obtained. Note that few applications of the variational iteration method to solve optimal control have been reported in the literature. [37] used the variational iteration method to solve the Ricatti differential equation to find the finite-time CLOC, i.e. under a state feedback form, for a linear system with quadratic performance index, known as the linear quadratic regulator (LQR). The same problem has been studied by Olotu and Adekunle [38]. Using the variational iteration method, Kucuk [39] proposed an active optimal control of the Korteweg and de Vries equation, that minimizes a quadratic functional, by considering the direct control parameterization approach.

The main contribution of the present paper consists in using the variational iteration method to solve iteratively the two-point boundary-conditions (Hamilton equations with the boundary conditions) and the unknown initial conditions of the costate variables can be simply determined by solving a set of nonlinear algebraic equations, which makes this approach an interesting alternative to the shooting method. In addition, although Hamilton equations are sufficient and necessary optimality conditions, the proposed approach provides the global solution. The choice of the variational iteration method is motivated by the fact that this method provides the solution of the differential equations even with time-variant parameters, in terms of a rapidly convergent infinite series, using correction functionals. Thus, an iterative procedure allows to overcome the instability of the direct numerical integration of Hamilton equations. The design method is illustrated by application examples that show the different possible forms (analytical, approximate analytical and numerical) of the optimal control that can be carried out following the proposed design approach.

The structure of the article is as follows. Section 2 is devoted to Pontryagin's minimum principle used for solving optimal control problem. Section 3 is dedicated to the proposed design approach to solve an open-loop optimal control problem based on the variational iteration method. The convergence of the method is demonstrated in this section. Section 4 illustrates the proposed design approach by some application examples. Finally, a conclusion ends the article.