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# Pullback attractors for the non-autonomous coupled suspension bridge equations

Jum-Ran Kang

*Department of Mathematics, Dong-A University, Busan 604-714, Republic of Korea*

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ABSTRACT

In this paper, we consider the pullback $\mathcal{D}$-attractors for the non-autonomous coupled suspension bridge equations when external terms are unbounded in a phase space.



## 1. Introduction

In this paper, we consider the following non-autonomous system which describes the vibrating beam equation coupled with a vibrating string equation:

$$\begin{cases} u_{tt} + \alpha u_{xxxx} + \delta_1 u_t + k(u - v)^+ + g_1(u) = f_1(x,t), \quad \text{in } [0,L] \times \mathbb{R}^+, \\ v_{tt} - \beta v_{xx} + \delta_2 v_t - k(u - v)^+ + g_2(v) = f_2(x,t), \quad \text{in } [0,L] \times \mathbb{R}^+, \\ u(0,t) = u(L,t) = u_{xx}(0,t) = u_{xx}(L,t) = 0, \quad t \geqslant \tau, \\ v(0,t) = v(L,t) = 0, \quad t \geqslant \tau, \\ u(x,\tau) = u_0(x), \quad u_t(x,\tau) = u_1(x), \quad x \in [0,L], \\ v(x,\tau) = v_0(x), \quad v_t(x,\tau) = v_1(x), \quad x \in [0,L], \end{cases} \tag{1.1}$$

where the first equation of (1.1) represents the vibration of the roadbed in the vertical direction and the second equation describes that of the main cable from which the roadbed is suspended by the tie cables (see [1]). $k > 0$ denotes the spring constant of the ties, $\alpha > 0$ and $\beta > 0$ are the flexural rigidity of the structure and coefficient of tensible strength of the cable, respectively. $\delta_1,\ \delta_2 > 0$ are constants, $f_1,\ f_2$ are time-dependent external forces, and

$$u^+ = \begin{cases} u, & \text{if } u > 0, \\ 0, & \text{if } u \leqslant 0. \end{cases}$$

The suspension bridge equations were presented by Lazer and McKenna as new problems in the field of nonlinear analysis [7]. For the autonomous problem corresponding to (1.1), there are many classical results. We refer the reader to [1,2,7,9–13,18] and references therein. Ma and Zhong [11] firstly investigated the existence of global attractors of the weak solutions for the suspension bridge equations. Zhong et al. [18] showed the existence of the strong solutions and the strong global attractors for the suspension bridge equations using the condition (C) introduced in [8] and combining with technique of energy estimates. Ma and Zhong [10] proved the existence of the global attractors of the weak solutions for the coupled system of suspension bridge equations. Ma and Zhong [12] proved the existence of strong solutions and global attractors for the

*E-mail address:* pointegg@hanmail.net

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coupled suspension bridge equations. Recently, Ma and Wang [9] showed the existence of pullback attractors for the coupled suspension bridge equations. We intend to investigate the dynamical behavior of the non-autonomous coupled suspension bridge equations.

Non-autonomous systems are also of great importance and interest as they appear in many applications in the natural science. The first attempts to extend the notion of global attractor to the non-autonomous case led to the concept of the uniform attractor (see Chepyzhov and Vishik [4]). The existence of uniform attractors relies on some compactness property of the solution operator associated with the system. But in some non-autonomous systems the trajectories can be unbounded when time increases to infinity; the classical theory of uniform attractors is not applicable in such systems. Hence, a different approach has been considered (see [5,6]) for the non-autonomous case. The global attractor is defined as a parameterized family of $A(\sigma)$, which attracts the solutions of the system from $-\infty$. This means that the initial instant of time goes to $-\infty$ and the final time remains fixed. We call such attractors as pullback attractor. In order to obtain the existence of pullback attractors, we always need to show the existence of a family of compact sets, which is pullback absorbing for the process associated with the systems. Caraballo et al. presented the concept of the pullback $\mathcal{D}$-attractors in [3], and obtained the abstract results verifying the existence of pullback $\mathcal{D}$-attractors, moreover, they applied their abstract results into the non-autonomous Navier–Stokes equation. Wang and Zhong [17] proved the pullback $\mathcal{D}$-attractors for non-autonomous sine–Gordon equations. Park and Kang [14] studied the pullback $\mathcal{D}$-attractors for non-autonomous suspension bridge equations with a more general external force in the space $H_0^2(\Omega)\times L^2(\Omega)$. Motivated by the ideas of [3,9,17], we study the existence of the pullback $\mathcal{D}$-attractors for the non-autonomous coupled suspension bridge equations with the more general external force. The nonlinear functions $g_1, g_2 \in C^2(\mathbb{R},\mathbb{R})$ satisfy the following assumptions:

$$\liminf_{|s|\to\infty}\frac{g_1(s)}{s}\geqslant 0,\quad \liminf_{|s|\to\infty}\frac{g_2(s)}{s}\geqslant 0; \tag{1.2}$$

$$|g_1(s)|,\ |g_2(s)|\leqslant C_0(1+|s|^p),\quad \forall p\geqslant 1, \tag{1.3}$$

$$\liminf_{|s|\to\infty}\frac{sg_1(s)-CG_1(s)}{s^2}\geqslant 0,\quad \liminf_{|s|\to\infty}\frac{sg_2(s)-CG_2(s)}{s^2}\geqslant 0, \tag{1.4}$$

where constants $C, C_0 > 0$, and

$$G_1(s)=\int_0^s g_1(\tau)d\tau,\quad G_2(s)=\int_0^s g_2(\tau)d\tau.$$

Let $\Omega=[0,L]$. With the usual notation, we denote

$$D(A)=H^2(\Omega)\cap H_0^1(\Omega),$$

$$D(A^2)=\{u\in H^2(\Omega)\,|\,A^2u\in L^2(\Omega),\ u(0)=u(L)=u_{xx}(0)=u_{xx}(L)=0\},$$

where $A=-\frac{\partial^2}{\partial x^2},\ A^2=\frac{\partial^4}{\partial x^4}$. And we introduce some spaces $H,\ V$ which are used throughout the paper, that is

$$H=L^2(\Omega)\times L^2(\Omega),\quad V=D(A)\times H_0^1(\Omega),$$

and endow space $H$ with the usual scalar product and norm, $(\cdot,\cdot),\ |\cdot|$, namely, for any $u=(u^1,u^2),\ v=(v^1,v^2)$, denote

$$(u,v)=\int_\Omega(u^1v^1+u^2v^2)dx,\quad |u|^2=|u^1|_{L^2}^2+|u^2|_{L^2}^2.$$

Clearly, we can also define the scalar product $((\cdot,\cdot))$ and norm $||\cdot||$ in $V$, i.e.,

$$((u,v))=\int_\Omega(u^1_{xx}v^1_{xx}+u^2_xv^2_x)dx,\quad ||u||^2=|u^1_{xx}|_{L^2}^2+|u^2_x|_{L^2}^2.$$

It is obvious that $H,\ V$ are Hilbert spaces, and $V\subset H=H^*\subset V^*$, here $H^*,\ V^*$ are the dual of $H,\ V$, respectively, and each space is dense in the following one and the injections are continuous.

For simplicity, we introduce two symbols $E_0$ and $E_1$:

$$E_0=V\times H,\quad E_1=(D(A^2)\times D(A))\times V.$$

Now, consider the following eigenvalue problems:

$$\begin{cases}-v_{xx}=\lambda v, & x\in\Omega,\\ v(0)=v(L)=0.\end{cases} \tag{1.5}$$

Let $\lambda_1$ be the first eigenvalue of (1.5), corresponding eigenfunction $\phi_1(x)$ is positive on $\Omega$. It is easy to know that $\lambda_1^2$ is the first eigenvalue in the following problems: