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Explicit doubly periodic soliton solutions for the (2+1)-dimensional Boussinesq equation ☆

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ABSTRACT

In this paper, (2+1)-dimensional Boussinesq equation is investigated. New explicit two soliton solution, periodic solitary wave solution and doubly periodic soliton solution are obtained by using special transformation of unknown function and three wave method with the aid of Maple.

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1. Introduction

In recent years, the studies of the exact solutions for the nonlinear evolution equations have attracted much attention to many mathematicians and physicists [1]. Many authors are interested in seeking soliton-like solution, because the waveforms can be changed in different mechanisms and it usually has travelling wave solutions [2–7].

As we all known that two-dimensional Boussinesq equation is introduced to describe the propagation of gravity waves on the surface of water, in particular the head-on collision of oblique waves. This equation combines the two-way propagation of the classical Boussinesq equation with the (weak) dependence on a second spatial variable, as occurs in the two-dimensional Korteweg–de Vries (2D KdV) equation [8]. In recent years, A series of solutions for the (2+1)-dimensional Boussinesq equation were obtained by using different methods. For example: Senthilvelan [9] studied the travelling wave solutions for (2+1)-dimensional Boussinesq equation by homogeneous balance method and explored certain new solution of the equation. Chen et al. [10] studied the (2+1)-dimensional Boussinesq equation by generalized transformation in homogeneous balance method and combined certain new solitary wave solutions, periodic wave solutions and the combined formal solitary wave solutions and periodic wave solutions of the equation. El-Sayed and Kaya [11] studied the (2+1)-dimensional Boussinesq equation by considering the decomposition scheme and obtained the exact solitary-wave solutions of the equation for the initial conditions without using any classical transformations and its numerical solutions are constructed without using any discretization technique. Abdel Rady et al. [12] studied the soliton solutions for (2+1)-dimensional Boussinesq equation by the repeated homogeneous balance method and obtained successfully many new exact travelling wave solutions. Recently, It is shown that exact and general solitary-wave, two-soliton and resonant solutions are obtained from the Hirota bilinear form of the equation [13–15]. We have seen that, although this equation can be written in bilinear form, it does not posess the general three-soliton solutions, comfirming the analysis of Hietarinta [16].

In this work, in order to search the influence of initial solution v_0 to multiple soliton solutions, we will discuss the (2+1)-dimensional Boussinesq equation by using the special transformation of unknown function and three wave method [17]. New explicit two soliton solution, periodic solitary wave solution and doubly periodic soliton solution are obtained.

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2. New two soliton and doubly periodic soliton solutions

We consider (2+1)-dimensional Boussinesq equation

$$u_{tt} - u_{xx} - u_{yy} - (u^2)_{xx} - u_{xxxx} = 0. \quad (2.1)$$

In this section, New explicit two soliton, periodic solitary wave solution and doubly periodic soliton solution are obtained by using the special transformation of unknown function and three wave method with the aid of the bilinear method.

We make transformation of unknown function

$$u(x, y, t) = 3v(x, y, t), \quad (2.2)$$

Substituting (2.2) into (2.1)we get

$$v_{tt} - v_{xx} - v_{yy} - 3(v^2)_{xx} - v_{xxxx} = 0, \quad (2.3)$$

Obviously, an arbitrary constant v_0 is the solution of (2.3), We suppose the solution of (2.3) as follows

$$v = v_0 + 2(\ln F)_{xx}, \quad (2.4)$$

where $F = F(x, y, t)$ is unknown real function.

Substituting (2.4)into (2.3), we obtain the bilinear equation as follows

$$(D_t^2 - (1 + 6v_0)D_x^2 - D_y^2 - D_x^4)F \cdot F = 0, \quad (2.5)$$

where, we let $v_0 = -1/6$,(2.5) is changed to the bilinear form

$$(D_t^2 - D_y^2 - D_x^4)F \cdot F = 0, \quad (2.6)$$

where

$$D_x^m D_y^n D_z^r D_t^s = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial y} - \frac{\partial}{\partial y'}\right)^n \left(\frac{\partial}{\partial z} - \frac{\partial}{\partial z'}\right)^r \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^s \Bigg|_{x=x', y=y', z=z', t=t'}.$$

With regard to Eq. (2.6), using three wave method, we choose the test function in the form

$$F = e^{a_1 x + b_1 y + c_1 t} + f\cos(a_2 x + b_2 y) + g\cosh(a_3 x + b_3 y + c_3 t) + he^{-a_1 x - b_1 y - c_1 t}, \quad (2.7)$$

where $a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_3, f, g$ and h are real constants to be determined.

Substituting (2.7) into (2.6) yields an algebraic equation of $e^{j(a_1x+b_1y+c_1t)}$, $e^{j(a_1x+b_1y+c_1t)}cos(a_2x + b_2y)$, $e^{j(a_1x+b_1y+c_1t)}\cosh(a_2x + b_2y)$. Equating all coefficients to zero, we get a set of algebraic equation for a_1, a_2, a_3, $b_1, b_2, b_3, c_1, c_3, f, g, h$,

$$\begin{cases} 4g^2a_3^4 + 16a_1^4h + 4f^2a_2^4 + 4b_1^2h - 4c_1^2h - g^2c_3^2 + g^2b_3^2 - f^2b_2^2 = 0, \\ -2b_1gb_3 + 2c_1gc_3 - 4ga_3^3a_1 - 4a_1^3ga_3 = 0, \\ 4ga_3^3fa_2 - 4fa_2^3ga_3 + 2fb_2gb_3 = 0, \\ 4fa_2^3ha_1 - 4ha_1^3fa_2 - 2fb_2hb_1 = 0, \\ -c_1^2hf - fb_2^2h - 6fa_2^2ha_1^2 + fa_2^4h + b_1^2hf + a_1^4hf = 0, \\ 2b_1fb_2 + 4a_1^3fa_2 - 4fa_2^3a_1 = 0, \\ -fb_2^2g - 6fa_2^2ga_3^2 + fa_2^4g - gc_3^2f + ga_3^4f + gb_3^2f = 0, \\ -2gc_3hc_1 + 2gb_3hb_1 + 4ga_3^3ha_1 + 4ha_1^3ga_3 = 0, \\ -c_1^2hg + gb_3^2h - gc_3^2h + b_1^2hg + ga_3^4h + 6ga_3^2ha_1^2 + a_1^4hg = 0, \\ a_1^4g + gb_3^2 + 6a_1^2ga_3^2 - gc_3^2 - c_1^2g + b_1^2g + ga_3^4 = 0, \\ fa_2^4 + b_1^2f - c_1^2f - 6a_1^2fa_2^2 - fb_2^2 + a_1^4f = 0. \end{cases} \quad (2.8)$$

Solving Eqs. (2.8) with the aid of Maple, we obtain many solutions for the determined parameters, according to the physical meaning, we only present three interesting results as follows

Case 1:

$$f = 0, \quad b_3 = 0, \quad c_1 = \frac{2a_1a_3(a_1^2 + a_3^2)}{c_3}, \quad h = \frac{g^2(4a_3^4 - c_3^2)}{4(6a_1^2a_3^2 + a_3^4 + c_3^2 - 3a_1^4)}, \quad b_1^2$$
$$= (-a_3^4c_3^2 - 6a_1^2a_3^2c_3^2 + c_3^4 - a_1^4c_3^2 + 4a_1^2a_3^6 + 8a_1^4a_3^4 + 4a_1^6a_3^2)c_3^{-2}, \quad (2.9)$$

where a_1, a_3 and c_3 are free parameters, substituting Eq. (2.9) into Eq. (2.7), with the aid of Eq. (2.2), especially, in the case of $h = 1$, we obtain two soliton solution of the (2+1)-dimensional Bq equation: