Applied Mathematics and Computation 219 (2013) 8910–8922

Contents lists available at SciVerse ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Detecting changes in the basin of attraction of a dynamical system: Application to the postural restoring system

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ARTICLE INFO

Keywords:
Modelling
Curve fitting
Dynamical system
Human balance

ABSTRACT

A method that provides a three-dimensional representation of the basin of attraction of a dynamical system from experimental data was applied to the problem of dynamic balance restoration. The method is based on the density of the data on the phase space of the system under study and makes use of modeling and numerical curve fitting tools. For the dynamical system of balance restoration, the shape and the size of the basin of attraction depend on the dynamics of the postural restoring mechanisms and contain important information regarding the biomechanical, as well as the neuromuscular condition of the individual. The aim of this work was to examine the ability of the method to detect, through the observed changes in the shape and/or the size of the calculated basins of attraction, (a) the inherent differences between different systems (in the current application, postural restoring systems of different individuals) and (b) induced changes in the same system (the postural restoring system of an individual). The results of the study confirm the validity of the method and furthermore justify its robustness.

1. Introduction

1.1. Obtaining the basin of attraction of a dynamical system

The problem of estimating the geometry of the basin of attraction of a dynamical system from experimental data is fundamental and challenging both from the theoretical and the applied point of view.

Analytical methods can be found in the literature, obviously limited to dynamical systems described by known, analytically manageable functional forms [1–10]. Numerical procedures have also been proposed for the calculation of the basin boundaries from first principles [9,11,12]. Methods such as the so-called cell mapping method [13] have been used. There has also been work on estimating the basin of attraction from experimental data [14–16] however in general such studies are concerned with the determination of the basin's lower/upper limits or its boundaries [17].

An interesting method for numerically determining the basin of attraction from experimental data can be found in [2]. Using as application a driven two-well magneto-mechanical oscillator, the basins of attractions for the system are obtained by an ensemble of initial conditions generated by switching between stochastic and deterministic excitation.

In our previous work [18], we presented a method of numerically estimating the shape and the size of the basin of attraction from the density of the experimental data. This method uses numerical optimization and data modeling tools to obtain

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0096-3003/$ - see front matter © 2013 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.amc.2013.03.030

analytical curves that describe both the contours and the boundary of the basin. Finally a three-dimensional function describing the basin of attraction is calculated.

The present work presents.

- a modification of the previous method [18], providing the three-dimensional basin of attraction in the form of a three-dimensional potential well and
- results of the application of the modified method to the study of the detection of changes in the geometry of the basin of attraction.

1.2. The application: the dynamic process of balance restoration after perturbation from the vertical

Given a sufficiently small perturbation, the human body has the ability to regain balance, following a complicated path back to the position of quiet vertical stance [19,20]. For vertical balance to be maintained, a variety of possible dynamics across different joints and body parts is recruited and a number of motor and sensory pathways contribute to its control [21]. Quiet stance and the maintenance of vertical balance is therefore an important motor function in humans.

The postural restoring response is a function of body morphology, muscular strength, and neurological condition; it can be affected by a number of factors, such as for example age, specific training, medication, impaired cognition, visual impairment, or even nutritional deficiencies. Injuries can also importantly affect balance, as the movement patterns leading to balance are changed so as to protect the injured area [22].

Quiet stance is rather a dynamic than a static phenomenon [23,24], as balance in quiet vertical stance does not imply motionless stability. The body's center of gravity oscillates spontaneously and continuously at low amplitude, a phenomenon known as postural sway [19,25,26]. Postural sway, still a matter of scientific discussion, is indistinguishable from correlated noise [27]. It has been suggested [23] that it is the body's clever strategy to maintain balance in vertical stance, instead of the result of an "imperfectness" of the biological system.

Although there is a lot of discussion, the integration of several mechanisms have been proposed, examples being vestibular sensing of head movement and gravitational orientation, visual sensing of self-movement, and proprioceptive sensing of ankle movement [21,28]. Two distinct re-stabilization mechanisms have been introduced, together with their combination: the "ankle strategy" and the "hip strategy" [23,24,29–33]. It seems that the postural system reduces the unknown, high number of the degrees of freedom of the stabilizing mechanisms by compressing them into these two muscular synergies at the neuromuscular level [24,32]. Static stability regions with respect to ankle and hip joints have been proposed [34], while the authors in [35] used a concept based on the mechanical limit of vertical stance with respect to the angles of the ankle and the hip, called stability cone [18].

Tools from non linear dynamics have been found to be very useful in understanding the complex behavior of vertical balance [18–20,22,24,25,36]. It has been furthermore shown that the relative motion of the ankles and the hips in postural oscillations exhibit typical hallmarks of dynamical systems, including relaxation time after perturbation [24,37]. From the point of view of dynamical systems, the difficult task of understanding the complex mechanism of the recuperation and maintenance of vertical balance can be facilitated by reducing the system's dimensionality [18,24,25].

The present study is based on a dynamical systems model [18,22,24,36] that does not make use of the individual components of the human body, or the non-linear interactions between them. Instead, it considers the process of dynamically regaining balance as movement inside a basin of attraction [18,22,24] that corresponds to the set of the correctable angles that an individual can lean to and still and reverse the motion so as to regain vertical stance (prevent falling). The important question of how best to stabilize the vertical position is this way transformed into the question of how to achieve a wider basin of attraction, with a stronger attractor at vertical stance.

The method [18] numerically calculates the basin of attraction that corresponds to an individual from experimental data, with the aim to demonstrate that the biomechanical differences and acquired asymmetries in the body's alignment are reflected as differences in the shape and the size of the basin of attraction of human balance. The method used in the present study can be used as an important and fundamental tool for the detection of changes of the dynamics of the system under study [18,22,24,36] and has potential applications that are not limited to the area of human balance, but can be extended to any other dynamical system where an estimation of its basin of attraction from experimental data is desired.

2. Theory/calculation

In what follows we provide a brief introduction to the model previously presented in [18,24], see also [22,36]:

2.1. The model on the phase space of (θ_x, θ_y)

Denoting as F_z the vertical component of the resultant ground reaction force and F_x, F_y its components on the x (anteroposterior) and y (mediolateral) directions respectively, then the model [24] assumes as only variables the two angles $(\theta_x, \theta_y) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, where $\theta_x \equiv tan^{-1}(F_x/F_z)$ and $\theta_y \equiv tan^{-1}(F_y/F_z)$.