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Strong form Meshless Implementation of Taylor Series Method

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ABSTRACT

A new meshless approach is investigated by using Taylor series expansion and technique of differential transform method, which is called Meshless Implementation of Taylor Series Method (MITSM). In particular, Strong form Meshless Implementation of Taylor Series Method (SMITSM) is studied in this paper. Then, the basis functions are used to solve a 1D second-order ordinary differential equation and 2D Laplace equation by using the SMITSM. Comparisons are made with the analytical solutions and results of Symmetric Smoothed Particle Hydrodynamics (SSPH) method. We also compared the effectiveness of the SMITSM and SSPH method by considering various particle distributions, nonhomogeneous terms and number of terms in the basis functions. It is observed that the MITSM has the conventional convergence properties and, at the expense of CPU time, yields smaller L_2 error norms than the SSPH method, especially in the existence of nonsmooth nonhomogeneous problems.

1. Introduction

Meshless smoothed particle hydrodynamics (SPH) method, proposed by Lucy [1] to study three-dimensional (3D) astrophysics problems, has been successfully applied to analyze transient fluid and solid mechanics problems. However, it has two shortcomings such as inaccuracy at particles on the boundary and tensile instability. Many techniques have been developed to alleviate these two deficiencies among which are corrected smoothed particle method (CSPM) [2,3], reproducing kernel particle method (RKPM) [4–6] and modified smoothed particle hydrodynamics (MSPH) method [7–10]. The MSPH method has been successfully applied to study wave propagation in functionally graded materials [9], can capture the stress field near a crack-tip, and simulates the propagation of multiple cracks in a linear elastic body [10]. The SSPH method has been applied to 2D homogeneous elastic problem successfully [11]. On the other hand, the SSPH method [11,12] is more suitable for homogeneous boundary value problems, cannot be easily applicable to nonlinear problems, requires at least fourth order terms in basis functions for the buckling problems which increases the CPU time.

Motivated by the fact that the SSPH method may not yield accurate results for solving nonhomogeneous problems due to its underlying formulation (e.g., see [13]), an alternative approach is investigated especially for nonhomegenous problems. To this end, based on the Taylor series expansion (TSE) and employing the technique of differential transform method (DTM), a new meshless approach called Meshless Implementation of Taylor Series Method (MITSM) is presented in this paper. Although both of the SSPH method and MITSM depend on TSEs, the main difference between these two approaches is as follows: the SSPH method calculates the value of the solution at a node by using the values of the solution at the other nodes and then substitute it into the governing differential equation; thus, nonhomogeneous terms in the governing differential equation are also evaluated pointwise at the nodes. This approach results in approximation errors especially in the existence of nonsmooth nonhomogeneous terms. On the other hand, the proposed MITSM approach substitute the TSEs of the solution

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and nonhomogeneous term into the governing differential equation and then utilize exact recursive relations between the coefficients of the expansions of the solution and nonhomogeneous term; it yields improvement in accuracy that is verified by solving numerical examples in Section 4. The MITSM can be applied to arbitrary boundary geometries, nonlinear problems, and strong and weak formulations. In particular, Strong form Meshless Implementation of Taylor Series Method (SMITSM) is investigated in this paper, whose results are compared with the analytical solutions and solutions of the SSPH method. It is shown that the SMITSM has the conventional convergence properties and yields smaller L_2 error norms in numerical examples than the SSPH method, especially in the existence of nonsmooth nonhomogeneous terms.

2. Differential transform method (DTM)

In this study, the DTM technique are employed to develop the MITSM. It is noteworthy that when the DTM is applied to ordinary differential equations, it exactly coincides with the traditional Taylor series method [14] where applications of TSEs and DTM are presented in detail. The 3D differential transform of a function $q(x,y,z)$ is defined as follows

$$Q(k,h,m) = \frac{1}{k!\,h!\,m!}\left[\frac{\partial^{k+h+m} q(x,y,z)}{\partial x^k y^h z^m}\right]_{(0,0,0)} \tag{2.1}$$

where $q(x,y,z)$ is the original function and $Q(k,h,m)$ is the transformed function. The inverse differential transform of $Q(k,h,m)$ is given by

$$q(x,y,z) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\sum_{m=0}^{\infty} Q(k,h,m)x^k y^h z^m \tag{2.2}$$

Some of the fundamental theorems on differential transform can be found in [15–20].

3. Strong form Meshless Implementation of Taylor Series Method (SMITSM)

Formulations of the SMITSM for 2D problems are presented in this section. For a function $T(x,y)$ which has continuous derivatives up to the $(n+1)$th order, the value of $T(x,y)$ at a point $\xi = (x,y)$ located in the neighborhood of a point $x_i = (x_i, y_i)$ can be written through the DTM technique as follows

$$T_i(x,y) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} U_i(k,h)(x-x_i)^k (y-y_i)^h \tag{3.1}$$

By introducing the matrices $\boldsymbol{P}(\boldsymbol{x},\xi)$ and $\boldsymbol{U}_i$, Eq. (3.1) can be cast into the following form

$$T_i(x,y) = \boldsymbol{P}(\boldsymbol{x},\xi)\boldsymbol{U}_i \tag{3.2}$$

where

$$\boldsymbol{P}(\boldsymbol{x},\xi) = \left[(x-x_i)^0(y-y_i)^0, (x-x_i)^1(y-y_i)^0, (x-x_i)^0(y-y_i)^1, \ldots, (x-x_i)^k(y-y_i)^h\right], \tag{3.3}$$

$$\boldsymbol{U}_i = [U_i(0,0), U_i(1,0), U_i(0,1), U_i(2,0), U_i(0,2), U_i(1,1), \ldots, U_i(k,h)]^T \tag{3.4}$$

Elements of $\boldsymbol{U}_i$ are unknown variables that are defined by

$$U_i(k,h) = \frac{1}{k!\,h!}\left[\frac{\partial^{k+h} T_i(x,y)}{\partial x^k \partial y^h}\right]_{(x_i,y_i)} \tag{3.5}$$

If both sides of Eq. (3.2) are multiplied by the kernel function $W(\xi,\boldsymbol{x})$, then we obtain

$$W(\xi,\boldsymbol{x})T_i(x,y) = W(\xi,\boldsymbol{x})\boldsymbol{P}(\boldsymbol{x},\xi)\boldsymbol{U}_i \tag{3.6}$$

In the compact support of $W(\xi,\boldsymbol{x})$ associated with the point $\boldsymbol{x_i} = (x_i,y_i)$, let N_g denote the number of particles, e.g., see Fig. 1.

If Eq. (3.6) is rewritten for all particles in the compact support domain shown in Fig. 1, by summing each side of these equations over these particles we get

$$\sum_{g=1}^{N_g} W(\xi_g,\boldsymbol{x_i})T_i(\xi_{\boldsymbol{g}}) = \sum_{g=1}^{N_g} \boldsymbol{W}(\xi_{\boldsymbol{g}},\boldsymbol{x_i})\boldsymbol{P}(\xi_{\boldsymbol{g}},\boldsymbol{x_i})\boldsymbol{U}_i \tag{3.7}$$

In order to generate equations as many as the number of unknowns in $\boldsymbol{U}_i$, Eq. (3.7) is rewritten by replacing $W(\xi,\boldsymbol{x})$ with the following derivatives