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On spectral properties of dissipative fourth order boundary-value problem with a spectral parameter in the boundary condition

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ABSTRACT

In this article, we consider dissipative fourth order boundary-value problem in the Lim-4 case with the spectral parameter in the boundary condition. We use the maximal dissipative operator to construct a self-adjoint dilation of this operator and its incoming and outgoing spectral representations. Then, we determine the scattering matrix of dilation. We also construct a functional model of the maximal dissipative operator and define its characteristic function in terms of solutions of the corresponding fourth order equation. Theorem on the completeness of the system of eigenvector and associated vectors of this operator are proved.

1. Introduction

The study of problems involving parameter dependent systems is of great interest to a lot of numerous problems in physics and engineering. A boundary value problem with a spectral parameter in the boundary condition appears commonly in mathematical models of mechanics. There are a lot of studies about parameter dependent problems [1,2,5,8,16–18,22,25,35–39].

The study of fourth order boundary value problems has increased recently. In general, the results known for the second order ones do not necessarily hold for the corresponding fourth order problems. It is well known that fourth order boundary value problems are related to the theory of beam deflection; in particular, the transverse motion of a rotating beam with tip mass, such as a helicopter blade [1] or a bob pendulum suspended from a wire [2].

The present paper focuses on the fourth order boundary value problem given by (2.2)–(2.6) with a spectral parameter in the boundary condition. The boundary value problem (2.2)–(2.6) leads to questions as to whether the boundary problem has a countable number of eigenvalues and whether eigenfunctions and associated functions term a complete system in the Hilbert space $L_2[0,\infty)$. Non-selfadjoint singular boundary value problems with λ– independent boundary conditions have been investigated in [3,4,6,7,24,27].

The spectral analysis of non-selfadjoint (dissipative) operators is based on the ideas of the functional model and dilation theory, rather than on traditional resolvent analysis and Riesz integrals. Using a functional model of a non-selfadjoint operator as a principal tool, spectral properties of such operators were investigated in [3–8,32–34]. The functional model of non-selfadjoint dissipative operators plays an important role within both the abstract operator theory and its more specialized applications in other disciplines. The construction of functional models for dissipative operators, natural analogues of spectral decompositions for self-adjoint operators are based on Nagy–Foias dilation theory [30] and Lax–Phillips scattering theory [29]. Pavlov's approach [32–34] to the model construction of dissipative extensions of symmetric operators was followed

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by Allahverdiev in [3–8] and some others, and by the group of authors [9–11], where the theory of the dissipative Schrodinger operator on a finite interval was applied to the problems arising in semiconductor physics. In [12–15], Pavlov's functional model was extended to (general) dissipative operators which are finite dimensional extensions of a symmetric operator, and the corresponding dissipative and Lax–Phillips scattering problems were investigated in some detail. The abstract extension methods that are used in the analysis of λ– dependent boundary problems were developed in [16,17,19–21].

This paper is organized as follows. In Section 2, we consider the maximal dissipative fourth order differential operators acting on the Hilbert space $L_2[0,\infty)$. We construct a self-adjoint dilation of the dissipative fourth order differential operator. We present in Section 3 its incoming and outgoing spectral representations which makes it possible to determine the scattering matrix of the dilation according to the Lax and Phillips scheme [29]. Later, a functional model of the dissipative fourth order differential operator is constructed by Pavlov's methods [32–34], and we define its characteristic functions in terms of the Titchmars–Weyl function of a self adjoint operator. Finally, we prove a theorem on completeness of the system of eigenfunctions and associated functions of dissipative operators under consideration. While proving our results, we use the machinary and methods of [3–8].

2. Construction of the dissipative fourth order operators in the Lim-4Case

Let us consider the differential expression

$$l(y) = y^{(4)} + q(x)y, \quad 0 \leqslant x < +\infty, \tag{2.1}$$

where $q(x)$ is a real continuous function on $[0,\infty)$.

We denote by L_0 the closure of the minimal operator [31] generated by (2.1), and by D_0 its domain. Further, we denote by D the set of all functions $y(x)$ in $L_2[0,\infty)$ such that their first three derivatives are locally absolutely continuous in $[0,\infty), l(y) \in L_2[0,\infty)$; and D is the domain of the maximal operator L, and $L = L_0^*$ (see [31]).

Assume that $q(x)$ is given in such a way that the operator L_0 has defect index (4,4). Let $\upsilon_1(x)$, $\upsilon_2(x)$, $\upsilon_3(x)$ and $\upsilon_4(x)$ be four linearly independent solutions of the equation $l(y) = 0$ satisfying the conditions

$$\begin{aligned}
&\upsilon_1(0) = 1, \quad \upsilon_1'(0) = 0, \quad \upsilon_1''(0) = 0, \quad \upsilon_1'''(0) = 0,\\
&\upsilon_2(0) = 0, \quad \upsilon_2'(0) = 1, \quad \upsilon_2''(0) = 0, \quad \upsilon_2'''(0) = 0,\\
&\upsilon_3(0) = 0, \quad \upsilon_3'(0) = 0, \quad \upsilon_3''(0) = 1, \quad \upsilon_3'''(0) = 0,\\
&\upsilon_4(0) = 0, \quad \upsilon_4'(0) = 0, \quad \upsilon_4''(0) = 0, \quad \upsilon_4'''(0) = 1,
\end{aligned}$$

at $x = 0$, and let their Wronskian be equal to 1. Since L_0 has defect index (4,4), we have $\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4 \in L_2[0,\infty)$.

We have Green's formula

$$(Ly, z)_{L^2} - (y, Lz)_{L^2} = [y, \bar{z}]_\infty - [y, \bar{z}]_0$$

for every $y, z \in D$, where $[y,z]_x = [y'''(x)z(x) - y(x)z'''(x)] - [y''(x)z'(x) - y'(x)z''(x)] \quad (0 \leqslant x < \infty)$.

Now let us consider the boundary value problem

$$l(y) = \lambda y, \quad y \in D, \quad 0 \leqslant x < \infty, \tag{2.2}$$

$$\alpha_1 y'''(0) - \alpha_2 y(0) = \lambda\left(\alpha_1' y'''(0) - \alpha_2' y(0)\right), \tag{2.3}$$

$$y'(0) = y''(0) = 0, \tag{2.4}$$

$$(F_1)(\infty) + \gamma(F_2)(\infty) := \begin{pmatrix} [y, \upsilon_2]_\infty \\ [y, \upsilon_1]_\infty \end{pmatrix} + \gamma \begin{pmatrix} [y, \upsilon_4]_\infty \\ [y, \upsilon_3]_\infty \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{2.5}$$

$$\gamma := \begin{pmatrix} -h_1 & 0 \\ 0 & -h_2 \end{pmatrix}, \quad \text{Im } h_1 > 0, \quad \text{Im } h_2 > 0,$$

where λ is a complex spectral parameter, $\alpha_1, \alpha_2, \alpha_1', \alpha_2' \in \mathbb{R}$ and

$$\alpha := \begin{vmatrix} \alpha_1' & \alpha_1 \\ \alpha_2' & \alpha_2 \end{vmatrix} > 0.$$

We will adopt the following notation:

$$\begin{aligned}
&R_0(y) = \alpha_1 y'''(0) - \alpha_2 y(0),\\
&R_0'(y) = \alpha_1' y'''(0) - \alpha_2' y(0),\\
&R_1(y) = [y, \upsilon_2]_\infty - h_1 [y, \upsilon_4]_\infty,\\
&R_2(y) = [y, \upsilon_1]_\infty - h_2 [y, \upsilon_3]_\infty.
\end{aligned}$$