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Stability criteria for BAM neural networks with leakage delays and probabilistic time-varying delays ☆

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ABSTRACT

This paper is concerned with the stability criteria for bidirectional associative memory (BAM) neural networks with leakage time delay and probabilistic time-varying delays. By establishing a stochastic variable with Bernoulli distribution, the information of probabilistic time-varying delay is transformed into the deterministic time-varying delay with stochastic parameters. Based on the Lyapunov–Krasovskii functional and stochastic analysis approach, delay-probability-distribution-dependent sufficient conditions are derived to achieve the globally asymptotically mean square stable of the considered BAM neural networks. The criteria are formulated in terms of a set of linear matrix inequalities (LMIs), which can be checked efficiently by use of some standard numerical packages. Finally, a numerical example and its simulations are given to demonstrate the usefulness and effectiveness of the proposed results.

1. Introduction

Over the past few decades, dynamical behavior of neural networks has been studied much in science and technology area, such as signal processing, parallel computing, optimization problems, and so on [1,2]. This led to significant attraction of many researchers, like mathematicians, physicists, computer scientists and biologist. In this regard, Hopfield [3] modeled a continuous time-dynamical neural networks which contain n dynamic neural units (DNUs) by the implementation of analog RC (resistance capacitance) network circuit. Further, Marcus and Westervelt [4] introduced a time delay into the above model. This contributed to the increased attention on stability analysis of various kind of neural network models such as Hopfield neural networks, cellular neural networks, Cohen–Grossberg neural networks, and BAM neural networks [5–19]. It is well known that BAM is a type of recurrent neural network which was introduced by Kosko in 1988 [20], who generalized the single auto-associative Hebbian correlator to a two-layer pattern-matched hetero-associative circuit. Recently, BAM neural networks have attracted the attention of many researchers [21–29], because of its widely application in the field of pattern recognition, automatic control associative memory and image processing.

Naturally, time delays which cause the poor performance and instability of dynamic systems are commonly encountered in various physical, engineering and neural based systems. Many existing works investigated about neural networks with deterministic time-delay, but at the same time, latest literatures are concerned with neural networks with stochastic

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time-delay. For example, if some values of the delay are very large but the probabilities of its occurrence are very small. In this case, we do not get less conservative results when only the information of variation range of the time delay is considered. Thus, the stability analysis of dynamic neural networks with random time-delay deserves to receive much attention and has been studied in recent years [30–33]. The authors in [30] addressed the problem of delay-distribution-dependent state estimation for discrete-time stochastic neural networks with random delay. Similarly, other authors have also proposed the delay-distribution-dependent stability of stochastic discrete-time neural networks with randomly mixed time-varying delays in [31].

On the other hand, a typical time delay called as Leakage (or "forgetting") delay may exist in the negative feedback terms of the neural network system and it has a great impact on the dynamic behaviors of delayed neural networks. Therefore, the leakage delay in dynamical neural networks is considered as an important research topic in stability analysis [37–43]. In this perspective, Gopalsamy [35] investigated the stability analysis for the BAM neural networks with constant delay in the leakage term. Further, Peng [36] addressed the BAM neural networks with continuously distributed delays in the leakage terms and derived conditions for the existence and global attraction of periodic solutions via the Lyapunov functional approach. Following this work, the authors of [37,38] have studied the stability of BAM neural networks with fuzzy and impulsive effect. Likewise, other researchers have also investigated the problem of stability analysis for neural networks and nonlinear system by using LMI technique, Lyapunov–Krasovskii functional and free matrix inequality in [39–43]. To the best of our knowledge, none have worked on the issue of stability criteria for BAM neural networks with time-delays in the leakage term and probabilistic time-varying delays, till now.

Motivated by the above discussion, the main objective of this paper is to propose the stability criteria for BAM neural networks with leakage delay and probabilistic time-varying delay functions by using a combination of Lyapunov–Krasovskii functional with triple integral terms, stochastic stability theory, Jenson's inequality and free-weighting matrices. All the criteria are expressed in terms of LMIs. Finally a numerical example is given to show the effectiveness and significance of the proposed criterion.

Notations: $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the n-dimensional Euclidean space and the set of all $n\times n$ real matrices respectively. The superscript T denotes the transposition and the notation $X \geqslant Y$ (similarly, $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (similarly, positive definite). $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$ and $\Lambda = \{1, 2, \ldots, n\}$. $Pr\{\alpha\}$ means the occurrence probability of the event α. $\mathbb{E}\{x\}$ and $\mathbb{E}\{x|y\}$, respectively, mean the expectation of the stochastic variable x and the expectation of the stochastic variable x conditional on the stochastic variable y. $diag\{\cdots\}$ stands for a block diagonal matrix. The notation $*$ always denotes the symmetric block in one symmetric matrix. $\lambda_{min}(\cdot)$ and $\lambda_{max}(\cdot)$ denote the minimum and maximum eigenvalues of a given matrix.

2. Problem description and preliminaries

The delayed BAM neural networks can be described as follows:

$$\begin{cases} \dot{u}_i(t) = -a_i u_i(t-\rho_1) + \sum_{j=1}^{m} b_{ij}^1 \tilde{f}_j(v_j(t)) + \sum_{j=1}^{m} b_{ij}^2 \tilde{f}_j(v_j(t-\tau(t))) + I_i, \quad i = 1, \ldots, n, \\ \dot{v}_j(t) = -c_j v_j(t-\rho_2) + \sum_{i=1}^{n} d_{ij}^1 \tilde{g}_i(u_i(t)) + \sum_{i=1}^{n} d_{ij}^2 \tilde{g}_i(u_i(t-\sigma(t))) + J_j, \quad j = 1, \ldots, m \end{cases} \tag{1}$$

or be rewritten in the following vector–matrix form:

$$\begin{cases} \dot{u}(t) = -Au(t-\rho_1) + B_1\tilde{f}(v(t)) + B_2\tilde{f}(v(t-\tau(t))) + I, \\ \dot{v}(t) = -Cv(t-\rho_2) + D_1\tilde{g}(u(t)) + D_2\tilde{g}(u(t-\sigma(t))) + J, \end{cases} \tag{2}$$

where $u(t) = [u_1(t), u_2(t), \ldots, u_n(t)]^T \in \mathbb{R}^n$, $v(t) = [v_1(t), v_2(t), \ldots, v_n(t)]^T \in \mathbb{R}^m$ are neuron state vectors, $A = diag\{a_1, \ldots, a_n,\} > 0$, $C = diag\{c_1, \ldots, c_n,\} > 0$ are diagonal matrices with positive entries $a_i > 0$ and $c_i > 0$, B_1 and D_1 are the connection weight matrices, B_2 and D_2 are the delayed connection weight matrices, $\tilde{f}(v(t)) = [\tilde{f}_1(v_1(t)), \ldots, \tilde{f}_m(v_m(t))]^T$, $\tilde{g}(u(t)) = [\tilde{g}_1(u_1(t), \ldots, \tilde{g}_n(u_n(t)]^T$ denote neuron activation functions, $I = [I_1, I_2, \ldots, I_n]^T$ and $J = [J_1, J_2, \ldots, J_m]^T$ are external inputs, the leakage delays $\rho_1 \geqslant 0$, $\rho_2 \geqslant 0$ are constants, and $\tau(t)$ and $\sigma(t)$ are time-varying delays and satisfy

$$0 \leqslant \tau(t) \leqslant \tau, \quad 0 \leqslant \sigma(t) \leqslant \sigma,$$

where τ and σ are positive constants.

The initial condition of the system (2) are assumed to be

$$u(s) = \phi(s) \quad s \in [-\tau^*, 0], \quad v(s) = \psi(s) \quad s \in [-\sigma*, 0],$$

where $\tau^* = \max\{\rho_1, \tau\}$, $\sigma^* = \max\{\rho_2, \sigma\}$ and the norms are defined by