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Numeric solutions for the pantograph type delay differential equation using First Boubaker polynomials

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ABSTRACT

A numerical method is applied to solve the pantograph equation with proportional delay under the mixed conditions. The method is based on the truncated First Boubaker series. The solution is obtained in terms of First Boubaker polynomials. Also, illustrative examples are included to demonstrate the validity and applicability of the technique. The results obtained are compared by the known results.

1. Introduction

This study is concerned with a generalization of a functional differential equation known as the pantograph equation (with constant and variable coefficients) which contains a linear functional argument (with retarded and advanced cases or with proportional delays). Functional differential equations with proportional delays are usually referred to as pantograph equations. The name "pantograph" originated from the work of Ockendon and Tayler on the collection of current by the pantograph head of an electric locomotive [1,2].

In recent years, pantograph equations have been studied by many authors, who have investigated both their analytical and numerical aspects [8–10]. These equations are characterized by the presence of a linear functional argument and play an important role in explaining many different phenomena. In particular, they arise in industrial applications [11] and in studies based on biology, economy, control theory, astro-physics, nonlinear dynamic systems, cell growth and electro-dynamic, among others [1,2,11,12]. In addition, properties of the analytical and numerical solutions of pantograph equations have been investigated by several authors [1–5,13–17].

The basic motivation of this work, by considering the mentioned studies, is to develop a new numerical method, which is called as a matrix method, based on Boubaker polynomials [3–7] and collocation points for the approximate solution of pantograph equation. The mentioned First Boubaker polynomials were firstly by Boubaker et al. (2006) as guide for solving a one-dimensional formulation at heat transfer equation.

In this study, we will consider the pantograph equation with variable coefficients

$$y^{(m)}(x) = \sum_{j=0}^{J}\sum_{k=0}^{m-1} P_{jk}(x)y^{(k)}(\alpha_j x + \mu_j) + g(x) \tag{1}$$

Under the initial conditions

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$$\sum_{k=0}^{m-1} c_{ik} y^{(k)}(0) = \lambda_i, \quad i = 0, 1, \ldots, m-1. \tag{2}$$

where $P_{jk}(x)$ and $g(x)$ are analytic functions; $c_{ik}, \lambda_i, \alpha_j$ and μ_j are real or complex constants. The aim of this study is to get solution of the problem (1) and (2) as the truncated First Boubaker series defined by

$$y(x) = \sum_{n=0}^{N} \beta_n \mathrm{B}_n(x), \quad 0 \leqslant x \leqslant b < \infty \tag{3}$$

where $\mathrm{B}_n(x)$, $n = 0, 1, 2, \ldots$ denote the First Boubaker polynomials; $\beta_n, 0 \leqslant n \leqslant \mathbf{N}$ are unknown First Boubaker coefficients, and $\mathbf{N}$ is chosen any positive integer such that $\mathbf{N} \geqslant m$. Here the standard First Boubaker polynomials are defined by

$$\begin{cases} \mathrm{B}_0(x) = 1, \mathrm{B}_1(x) = x, \mathrm{B}_2(x) = x^2 + 2, \\ \mathrm{B}_m(x) = x\mathrm{B}_{m-1}(x) - \mathrm{B}_{m-2}(x) \quad \text{for} : m > 2 \end{cases}$$

and a monomial definition of these polynomials was established by Labiadh et al. [7]:

$$\mathrm{B}_n(x) = \sum_{p=0}^{\xi(n)} \left[\frac{(n-4p)}{(n-p)} \mathrm{C}_{n-p}^{p} \right] . (-1)^p . x^{n-2p} \tag{4}$$

where $\xi(n) = \lfloor \frac{n}{2} \rfloor$ is denotes the floor function.

2. Fundamental relations

Let us consider the pantograph Eq. (1) and find the matrix forms of each term in the equation. First we can convert the solution $y(x)$ defined by the truncated series (3) and its derivative $y^{(k)}(x)$ to matrix forms

$$y(x) = \mathbf{B}(x)\beta \text{ and } y^{(k)}(x) = \mathbf{B}^{(k)}(x)\beta, \quad k = 0, 1, 2, \ldots \tag{5}$$

where

$$\mathbf{B}(x) = [\mathrm{B}_0(x)\mathrm{B}_1(x) \cdots \mathrm{B}_N(x)], \quad \beta = [\, \beta_0 \quad \beta_1 \quad \ldots \quad \beta_N \,]^T$$

By using the expression (4) and taking $n = 0, 1, \ldots, \mathbf{N}$, we find the corresponding matrix relation as

$$\mathbf{B}(x) = \mathbf{X}(x)\mathbf{Z}^T \tag{6}$$

where

$$\mathbf{X}(x) = [1\, x \ldots x^N]$$

and if $\mathbf{N}$ is odd,

$$\mathbf{Z} = \begin{bmatrix} \varphi_{0,0} & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & \varphi_{1,0} & 0 & 0 & \ldots & 0 & 0 \\ \varphi_{2,1} & 0 & \varphi_{2,0} & 0 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \varphi_{N-1,\frac{N-1}{2}} & 0 & \varphi_{N-1,\frac{N-3}{2}} & 0 & \ldots & \varphi_{N-1,0} & 0 \\ 0 & \varphi_{N,\frac{N-1}{2}} & 0 & \varphi_{N,\frac{N-3}{2}} & \ldots & 0 & \varphi_{N,0} \end{bmatrix}$$

if $\mathbf{N}$ is even,

$$\mathbf{Z} = \begin{bmatrix} \varphi_{0,0} & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & \varphi_{1,0} & 0 & 0 & \ldots & 0 & 0 \\ \varphi_{2,1} & 0 & \varphi_{2,0} & 0 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \varphi_{N-1,\frac{N-2}{2}} & 0 & \varphi_{N-1,\frac{N-4}{2}} & \ldots & \varphi_{N-1,0} & 0 \\ \varphi_{N,\frac{N}{2}} & 0 & \varphi_{N,\frac{N-2}{2}} & 0 & \ldots & 0 & \varphi_{N,0} \end{bmatrix}$$

where

$$B_n(x) = \sum_{p=0}^{\xi(n)} \varphi_{n,p} x^{n-2p}, \quad n = 0, 1, \ldots, N, \quad p = 0, 1, \ldots, \left\lfloor \frac{n}{2} \right\rfloor.$$