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Time-delay estimation for nonlinear systems with piecewise-constant input

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ABSTRACT

We consider a general nonlinear time-delay system in which the input signal is piecewise-constant. Such systems arise in a wide range of industrial applications, including evaporation and purification processes and chromatography. We assume that the time-delays—one involving the state variables and the other involving the input variables—are unknown and need to be estimated using experimental data. We formulate the problem of estimating the unknown delays as a nonlinear optimization problem in which the cost function measures the least-squares error between predicted and measured system output. The main difficulty with this problem is that the delays are decision variables to be optimized, rather than fixed values. Thus, conventional optimization techniques are not directly applicable. We propose a new computational approach based on a novel algorithm for computing the cost function's gradient. We then apply this approach to estimate the time-delays in two industrial chemical processes: a zinc sulphate purification process and a sodium aluminate evaporation process.

1. Introduction

Time-delay dynamic systems have been an active area of research over the past two decades; see, for example, [1–10] and the references cited therein. Indeed, time-delays are inherent in many industrial processes, including evaporation processes [11], chromatography processes [12], distillation processes [13], and purification processes [14]. Such processes can be controlled by varying certain input variables—for example, flow rates, temperatures, and pressures. If the time-delays are known, then the problem of determining the optimal input variables (as functions of time) so that the total system cost is minimized is a so-called *optimal control problem*. Such problems can be solved numerically using well-known computational techniques [14–17].

In many situations, however, the time-delays are not known exactly. In this case, the delays first need to be estimated before optimal control techniques can be applied. Thus, delay estimation is a crucial issue and has attracted significant research attention over the past decade. The vast majority of delay estimation methods are only applicable to simple systems with linear dynamics and a single delay [18–25]. One of the few methods available for handling general nonlinear systems with multiple time-delays is the optimization-based approach developed in [26]. In this approach, the problem of estimating the time-delays is formulated as a dynamic optimization problem in which the cost function measures the discrepancy be-

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tween predicted output and observed system output at a set of sample times. Solving this optimization problem yields the delay values that best fit the given experimental data.

The optimization-based approach developed in [26] is designed for systems with state-delays rather than input-delays. For systems with input-delays, if the input function is smooth, then the system dynamics will be continuously differentiable with respect to the input-delays, and thus the approach proposed in [26] can be easily modified to estimate the input-delays in this case. Unfortunately, the input function is often non-smooth in practical applications. For example, in the evaporation process described in [11], the input function represents the solution flow rate, which must be kept constant and is only changed at 5 minute intervals to ensure process stability. Thus, the solution flow rate is a non-smooth piecewise-constant input function. The estimation method in [26] is not applicable in such situations.

With this motivation, we consider in this paper the time-delay estimation problem for nonlinear systems in which the input function is piecewise-constant. We assume that the system under consideration contains one state-delay and one input-delay, both of which are unknown and need to be estimated using experimental data. Since the input function is discontinuous, the estimation method in [26] is not applicable in this case. The purpose of this paper is to develop a new method for estimating the unknown time-delays. As with [26], we formulate the delay estimation problem as a dynamic optimization problem in which the cost function measures the least-squares error between predicted and observed system output. The main focus of the paper is on the derivation of a computational procedure for determining the gradient of the cost function. This procedure, which involves integrating an auxiliary impulsive system with instantaneous jumps forward in time, is far more complex than the procedure given in [26], which does not involve any jumps. Moreover, because of the discontinuous nature of the input function, the cost function's gradient does not exist at certain points. We propose a heuristic strategy for dealing this complication. This heuristic strategy can be combined with our gradient computation procedure to solve the estimation problem using standard nonlinear programming algorithms. We finally conclude the paper by showing that this approach can successfully estimate the time-delays in two large-scale chemical engineering systems.

2. Problem formulation

Consider the following nonlinear time-delay system:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{x}(t-\alpha), \boldsymbol{u}(t), \boldsymbol{u}(t-\beta)), \quad t \in [0, T], \tag{1}$$

$$\boldsymbol{x}(t) = \boldsymbol{\phi}(t), \quad t \leqslant 0, \tag{2}$$

where $T > 0$ is a given *terminal time*; $\boldsymbol{x}(t) = [x_1(t), \ldots, x_n(t)]^\top \in \mathbb{R}^n$ is the *state vector*; $\boldsymbol{u}(t) = [u_1(t), \ldots, u_r(t)]^\top \in \mathbb{R}^r$ is the *input vector*; α and β are unknown time-delays that need to be determined; and $\boldsymbol{f} : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^r \times \mathbb{R}^r \to \mathbb{R}^n$ and $\boldsymbol{\phi} : \mathbb{R} \to \mathbb{R}^n$ are given functions. Many dynamic processes in chemical engineering—for example, the distillation process described in [27]—can be modeled by Eqs. (1) and (2). We assume that $\boldsymbol{f}$ and $\boldsymbol{\phi}$ are continuously differentiable. We also assume that there exists a positive real number $L_1 > 0$ such that for all $\boldsymbol{x}', \boldsymbol{x}'' \in \mathbb{R}^n$ and $\boldsymbol{u}', \boldsymbol{u}'' \in \mathbb{R}^r$,

$$|\boldsymbol{f}(\boldsymbol{x}', \boldsymbol{x}'', \boldsymbol{u}', \boldsymbol{u}'')| \leqslant L_1(1 + |\boldsymbol{x}'| + |\boldsymbol{x}''| + |\boldsymbol{u}'| + |\boldsymbol{u}''|), \tag{3}$$

where $|\cdot|$ denotes the Euclidean norm. This assumption is standard in the control systems literature [14,16,28–30].

The output $\boldsymbol{y}(t)$ of system (1) and (2) is defined by

$$\boldsymbol{y}(t) = \boldsymbol{g}(\boldsymbol{x}(t)), \quad t \in [0, T], \tag{4}$$

where $\boldsymbol{g} : \mathbb{R}^n \to \mathbb{R}^q$ is a given continuously differentiable function.

We refer to α as the *state-delay* and β as the *input-delay*. The exact values of these delays are unknown; the only information we are given is that α lies within the interval $[\alpha_{\min}, \alpha_{\max}]$ and β lies within the interval $[\beta_{\min}, \beta_{\max}]$, where $\alpha_{\min} \geqslant 0$ and $\beta_{\min} > 0$. Thus, we have the following bound constraints:

$$\alpha_{\min} \leqslant \alpha \leqslant \alpha_{\max}, \tag{5}$$

$$\beta_{\min} \leqslant \beta \leqslant \beta_{\max}. \tag{6}$$

We assume that the input signal $\boldsymbol{u}$ is a given piecewise-constant function (this is the case in many engineering systems). Hence, $\boldsymbol{u}$ can be expressed as follows:

$$\boldsymbol{u}(t) = \boldsymbol{\sigma}^i, \quad t \in [t_{i-1}, t_i), \; i = 1, \ldots, p, \tag{7}$$

where $\boldsymbol{\sigma}^i \in \mathbb{R}^r, i = 1, \ldots, p$, are given vectors and $t_i, i = 0, \ldots, p$, are given time points such that

$$-\beta_{\max} = t_0 < t_1 < \cdots < t_p = T.$$

Eq. (7) can be rewritten as

$$\boldsymbol{u}(t) = \sum_{i=1}^{p} \boldsymbol{\sigma}^i \chi_{[t_{i-1}, t_i)}(t), \quad t \in [-\beta_{\max}, T], \tag{8}$$

where the characteristic function $\chi_{[t_{i-1}, t_i)} : \mathbb{R} \to \mathbb{R}$ is defined by