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Global convergence of a general filter algorithm based on an efficiency condition of the step

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ABSTRACT

In this work we discuss global convergence of a general filter algorithm that depends neither on the definition of the forbidden region, which can be given by the original or slanting filter rule, nor on the way in which the step is computed. This algorithm basically consists of calculating a point not forbidden by the filter from the current point. Assuming that this step must be efficient, in the sense that near a feasible non-stationary point the decrease in the objective function is relatively large, we prove the global convergence of the algorithm. We also discuss that such a condition is satisfied if the step is computed by the SQP or Inexact Restoration methods. For SQP we present a general proof of this result that is valid for both the original and the slanting filter criterion. In order to compare the performance of the general filter algorithm according to the method used to calculate the step and the filter rule regarded, we present numerical experiments performed with problems from CUTEr collection.

1. Introduction

In this work we shall discuss global convergence of filter methods for solving the nonlinear programming problem

$$\begin{array}{ll} \text{minimize} & f(x), \\ \text{subject to} & c_{\mathcal{E}}(x) = 0, \\ & c_{\mathcal{I}}(x) \leqslant 0, \end{array} \tag{1}$$

where the index sets $\mathcal{E}$ and $\mathcal{I}$ refer to the equality and inequality constraints, respectively. Let the cardinality of $\mathcal{E} \cup \mathcal{I}$ be m, and assume that the functions $f : \mathbb{R}^n \to \mathbb{R}, c_i : \mathbb{R}^n \to \mathbb{R}, i = 1, \dots, m$, are twice continuously differentiable and the constraints are qualified. The Jacobian matrices of $c_{\mathcal{E}}$ and $c_{\mathcal{I}}$ are denoted, respectively, by $A_{\mathcal{E}}$ and $A_{\mathcal{I}}$.

We define an infeasibility measure function $h : \mathbb{R}^n \to \mathbb{R}_+$ by

$$h(x) = \|c^+(x)\|, \tag{2}$$

where $\| \cdot \|$ is an arbitrary norm and $c^+ : \mathbb{R}^n \to \mathbb{R}^m$ is defined by

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$$c_i^+(x) = \begin{cases} c_i(x) & \text{if } i \in \mathcal{E}, \\ \max\{0, c_i(x)\} & \text{if } i \in \mathcal{I}. \end{cases}$$

Filter methods, introduced by Fletcher and Leyffer in their seminal paper [5], define a *forbidden region* by memorizing pairs $(f(x^j), h(x^j))$ chosen conveniently from former iterations and then avoid points in the regions

$$\mathcal{R}_j = \{x \in \mathbb{R}^n | f(x) \geqslant f(x^j) - \alpha h(x^j) \text{ and } h(x) \geqslant (1-\alpha)h(x^j)\}, \tag{3}$$

where $\alpha \in (0,1)$ is a given constant. A slightly different way of defining the domination rule, proposed initially by Chin [1], regards the regions as

$$\mathcal{R}_j = \{x \in \mathbb{R}^n | f(x) + \alpha h(x) \geqslant f(x^j) \text{ and } h(x) \geqslant (1-\alpha)h(x^j)\}. \tag{4}$$

The filter based on the rule (3) will be referred as *original filter* and the one based on (4) will be called *slanting filter*.

From now on, we simplify the notation, when appropriate, by using (f^j, h^j) to represent $(f(x^j), h(x^j))$. Fig. 1 shows the forbidden region in the plane $f \times h$. The pictures on the left and right illustrate the original and the slanting filter, respectively.

Global convergence of filter algorithms has been proved in several works. Under reasonable assumptions, Fletcher et al. [4] proved that the sequence generated by a filter-SQP algorithm based on the filter rule (3) has a stationary accumulation point. Chin and Fletcher [2] and Fletcher, Leyffer and Toint [6] proved global convergence of a slanting filter algorithm which computes the new iterates by sequential linear programming and sequential quadratic programming, respectively.

Filter techniques have also been applied to interior point methods [23], line search algorithms [11,24–26], nonsmooth convex constrained optimization [14], complementarity problems [15,16], systems of nonlinear equations [8] and unconstrained optimization [10].

The efficiency condition of the step considered in this work is based on that introduced by Gonzaga, Karas and Vanti [7], which presented a general globally convergent filter algorithm, using the original filter rule, that leaves the step computation separate from the main algorithm. They showed that any method for computing the step can be used, since the points generated must be acceptable for the filter and that near a feasible non-stationary point the reduction of the objective function is relatively large. For completeness, they showed that the Inexact Restoration method of Martínez and Pilotta [17,18] satisfies such a condition. Using the same ideas of [7], Ribeiro, Karas and Gonzaga [22] proved global convergence of filter methods under a weaker version of the efficiency condition introduced in [7] which are proved to be satisfied by SQP methods with the original filter rule.

Karas, Oening and Ribeiro [13] proposed a slanting filter method which uses Inexact Restoration for computing the step and, by assuming the same efficiency condition considered in [7], they proved a stronger result about stationarity, namely, that all accumulation points of the sequence generated by the algorithm are stationary.

In this work we prove that the general filter algorithm presented in [22] is globally convergent regardless of the filter criterion used, original or slanting. For proving the convergence, we assume that the step satisfies an efficiency condition of the step, stated below as Hypothesis H3. To fulfil our analysis, we present the proof that the step computed by the SQP method satisfies this condition. However, unlike [22], where this result is proved by considering the original filter, we do not take into account a particular filter rule in our proof, being valid for both the original and the slanting filter criterion. Moreover, we discuss an Inexact Restoration method that can also be applied to determine the step. In order to verify the efficiency of these algorithms, we present results of numerical experiments performed on some CUTEr problems.

The paper is organized as follows. The general filter algorithm and its convergence analysis are described in Section 2 and 3, respectively. In Section 4 we present the SQP method for computing the step and prove that it satisfies the Hypothesis H3. Finally, in Section 5, numerical experiments on CUTEr problems are presented.

2. The general algorithm

In this section we present a general filter algorithm that allows a great deal of freedom in the definition of the forbidden region and in the step computation. We also show that this algorithm is well-defined for both original and slanting criteria.

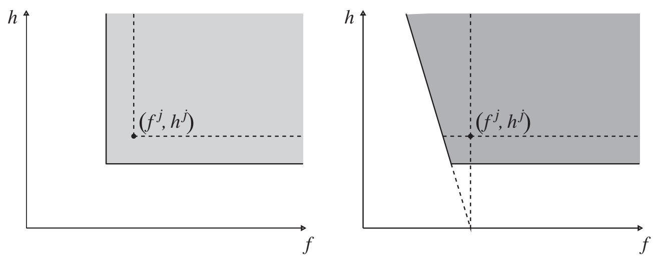

Fig. 1. Forbidden Region.