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Effect of magnetic field on the viscous fluid flow in a channel filled with porous medium of variable permeability

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ABSTRACT

The present work concerns the study of the fully developed flow in a channel of an incompressible, electrically conducting viscous fluid through a porous medium of variable permeability under the transverse applied uniform magnetic field. The variation of permeability is taken quadratic on the transverse direction and small. The Brinkman equation is used for flow through porous medium. Numerical expressions by applying Galerkin's method for the velocity and volumetric flow rate for two cases, Poiseuille and Couette flow are obtained. The influence of the various parameters like Hartmann number, permeability variation, etc. on the velocity profile and flow rate is discussed.

1. Introduction

The study of viscous fluid flow in porous media has been a topic of longstanding interest for researchers due to its numerous applications in many fields such as bio-mechanics, physical sciences and chemical engineering, etc. An arbitrary flow of viscous, incompressible fluid through a swarm of porous particles has many industrial and engineering applications, such as, flow through porous beds, in petroleum reservoir rocks, in flow sedimentation. Several researchers have considered the flow of viscous fluid past and through solids or porous bodies with different models. A number of theoretical and experimental models have been developed to describe fluid flow through porous medium.

Darcy's law as proposed by Henri Darcy [1], states that the rate of flow is proportional to pressure drop through a densely packed bed of fine particles, is one of the basic model that has been used extensively in the literature. This law can be expressed as

$$-\frac{\tilde{\mu}}{\tilde{k}}\tilde{\mathbf{v}} = \nabla\tilde{p}. \tag{1}$$

Brinkman [2] proposed a modification of Darcy's law for porous medium and provided an expression like

$$\tilde{\mu}_e\nabla^2\tilde{\mathbf{v}} - \frac{\tilde{\mu}}{\tilde{k}}\tilde{\mathbf{v}} = \nabla\tilde{p}, \tag{2}$$

where $\tilde{\mathbf{v}}$ is the velocity vector, $\tilde{p}$ is the pressure, $\tilde{k}$ is the permeability, $\tilde{\mu}$ is the fluid viscosity and $\tilde{\mu}_e$ is the effective viscosity of the fluid flowing in the porous medium.

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Nield and Bejan [3] published many of these extended models in their celebrated book. Hamdan [4] has reviewed the leading models of single-phase fluid flow through porous media and discussed the boundary conditions associated with these models.

Bali and Awasthi [5] have analyzed the effect of external magnetic field on blood flow in stenotic artery and they have taken the effect of magnetic field in transverse direction to blood flow, and considered the viscosity of blood as radial coordinate dependent and discussed the velocity resistance to the blood flow. A solution of partial differential equation representing irrotational flow in bispherical polar coordinates was evaluated by Deo and Tiwari [6].

Hamdan et al. [7] have analyzed Brinkman's equation taking permeability variation as a quadratic function of normalized transverse distance of the channel and evaluated an expression of velocity profile. Verma and Datta [8] have studied the flow of a viscous, incompressible, electrically conducting fluid with varying viscosity through a channel in the presence of a transverse magnetic field. They have taken viscosity as a function of distance from the mid section of the channel and obtained exact solutions for velocity. Varshney et al. [9] developed a mathematical model for the blood flow in stenosed artery in the presence of magnetic field and they have discussed laminar, incompressible, fully developed, non-Newtonian flow of blood in an artery having multiple stenosis numerically. Beg et al. [10] obtained a numerical solution for the viscous, incompressible, magnetohydrodynamic flow in a rotating channel comprising two infinite parallel plates and containing a Darcian porous medium under constant pressure gradient.

Hydrodynamic permeability of aggregates of porous particles with an impermeable core by applying cell methods was evaluated by Deo et al. [11]. They have reported some new results for calculated hydrodynamic permeability and theoretical values of Kozeny constant. Recently, Tiwari et al. [12] have studied the effect of magnetic field on the hydrodynamic permeability of a membrane built up by solid cylindrical particles covered by porous layer.

The purpose of this paper is to study the fully developed flow in a channel of an incompressible, electrically conducting viscous fluid through a porous medium of variable permeability under the transverse applied uniform magnetic field. The variation of permeability is taken quadratic on the transverse direction and small. The resulting equation has been solved numerically by using Galerkin method. Expressions for the velocity and volumetric flow rate for cases, Poiseuille and Couette flow are obtained. The influence of the various parameters on the velocity profile and flow rate is discussed.

2. Mathematical formulation of the problem

Let us consider the fully developed plane Poiseuille flow of an incompressible, electrically conducting viscous fluid through a porous medium of variable permeability in a channel. The upper and lower plates are at rest and the flow is driven by a constant pressure gradient (Fig. 1). The permeability variation in the channel is taken along the transverse direction, so it can be taken as $\tilde{k} = \tilde{k}(y)$, where y is a dimensionless variable as defined latter [8]. A transverse magnetic field of uniform intensity is applied. The Magnetic Reynolds number is assumed to be very small and there is no external electric field so that induced current is very small and hence, it can be neglected. Therefore, the governing equation (2) can be expressed as

$$\tilde{\mu}_e \nabla^2 \tilde{\mathbf{v}} - \frac{\tilde{\mu}}{\tilde{k}(y)} \tilde{\mathbf{v}} + \tilde{\mathbf{J}} \times \tilde{\mathbf{B}} = \nabla \tilde{p}, \tag{3}$$

where $\tilde{\mathbf{J}}$ is the electric current density and $\tilde{\mathbf{B}}$ is the magnetic induction vector of applied uniform magnetic field. As we have assumed that external electric field is absent and internal causes such as separation of charges or polarization do not give rise to induced electric field, therefore $\tilde{\mathbf{J}} = \tilde{\sigma}(\tilde{\mathbf{v}} \times \tilde{\mathbf{B}})$, where $\tilde{\sigma}$ is the electrical conductivity of fluid. Therefore, we conclude that Lorentz force $\tilde{\mathbf{F}} = \tilde{\mathbf{J}} \times \tilde{\mathbf{B}}$ and velocity $\tilde{\mathbf{v}}$ are collinear and in opposite directions, hence $\tilde{\mathbf{F}} = -\tilde{\sigma}\tilde{B}_0^2 \tilde{\mathbf{v}}$, where $\tilde{B}_0 = |\tilde{\mathbf{B}}|$.

Therefore, Eq. (3) now assume the following simplified form

$$\frac{d^2\tilde{u}}{d^2\tilde{y}} - \frac{\tilde{\mu}}{\tilde{\mu}_e} \frac{1}{\tilde{k}(y)} \tilde{u} - \frac{\tilde{\sigma}\tilde{B}_0^2}{\tilde{\mu}_e} \tilde{u} = \frac{1}{\tilde{\mu}_e} \frac{d\tilde{p}}{d\tilde{x}}. \tag{4}$$

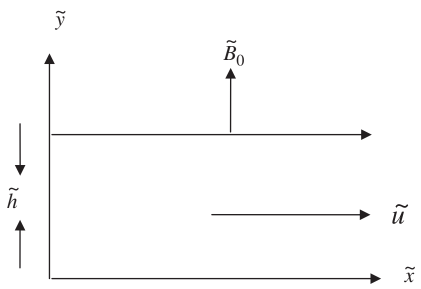

Fig. 1. Schematic diagram of the physical problem.