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Quasi-exact solutions of the dissipative Kuramoto–Sivashinsky equation

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ABSTRACT

The dissipative Kuramoto–Sivashinsky equation is studied. It is shown that this equation does not pass the Painlevé test and as consequence this equation is not integrable. Quasi-exact solution of the dissipative Kuramoto–Sivashinsky equation is given.

1. Introduction

Definition of the quasi-exact solution was introduced recently in Ref. [1]. This expansion of understanding exact solution allows us to have the approximate solution of nonlinear differential equation in the case when we cannot find exact solution.

The idea of quasi-exact solution completely reminds the approach of finding the numerical solution of boundary value problem.

The matter is when we apply numerical method for constructing difference solution we always consider other mathematical model that is close to model written in the differential form. However we look for numerical solution that is close to the solution of the original problem.

Our approach of finding quasi-exact solution similar to procedure of obtaining numerical solution. Specially our method is effective when we cannot find exact solution of nonlinear differential equation. In this case we look for quasi-exact solution of mathematical model that is close to initial model.

Let us apply our method for finding quasi-exact solution of the equation

$$u_t + uu_x + u_{xxxx} = 0. \tag{1.1}$$

We call this equation as the dissipative Kuramoto–Sivashinsky equation because (1.1) is the partial case of the famous Kuramoto–Sivashinsky equation [2–7]

$$u_t + uu_x + \alpha u_{xx} + \beta u_{xxx} + \delta u_{xxxx} = 0. \tag{1.2}$$

It is well known that Eq.(1.2) is not integrable equation by the inverse scattering transform but has some exact solutions in the case of the following conditions on parameters of equation [8,9]

$$\frac{\beta}{\sqrt{\alpha\delta}} = 0; \quad \pm\frac{12}{\sqrt{47}}; \quad \pm\frac{16}{\sqrt{73}}; \quad \pm 4. \tag{1.3}$$

Exact solutions of (1.2) were found in many papers (see, for a example, [10–20]).

In this paper we show that Eq.(1.1) has only simple rational solution and does not have any solitary wave solution. We obtain the quasi-exact solution of Eq.(1.1) that is solution of equation in the form

$$u_t + uu_x + u_{xxxx} = k^4 F(u), \tag{1.4}$$

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where k is the wave number for solution of quasi-exact solution for Eq.(1.1).

One can see that in the case $k \to 0$ Eq.(1.4) tends to Eq.(1.1). So we obtain that the exact solution of Eq.(1.4) corresponds to quasi-exact solution of Eq.(1.1).

Algorithm of finding quasi-exact solution coincides with the algorithm of obtaining exact solutions for nonlinear ordinary differential equation presented in papers [21–26]. The possibility of our approach for constructing quasi-exact solutions of nonlinear differential equations is one of the advantage by this algorithm in comparison with other methods.

2. The Painlevé analysis of the dissipative Kuramoto–Sivashinsky equation

The Painlevé test for nonlinear differential equations is powerful approach for testing integrable differential equation [27–31].

Let us study the nonlinear ordinary differential Eq. (1.1) taking the traveling wave solutions. Using

$$u(x,t) = y(z), \quad z = kx - \omega t. \tag{2.1}$$

From (1.1) we have after integration the following equation

$$k^4 y_{zzzz} + k y y_z - \omega y_z = 0. \tag{2.2}$$

The equation with the leading members corresponding to (2.2) takes the form [28]

$$k^4 y_{zzzz} + k y y_z = 0. \tag{2.3}$$

Substituting $y = a_0/z^p$ into Eq. (2.3) we have $(a_0, p) = (120k^3, 3)$. So, we have the first member of the solution expansion in the Laurent series in the form

$$y \simeq \frac{120k^3}{(z-z_0)^3} + \cdots \tag{2.4}$$

Substituting

$$y \simeq \frac{120k^3}{z^3} + a_j z^{j-3}, \tag{2.5}$$

into (2.3) again and equating the expression at first order a_j, we obtain the following Fuchs indices for solution of Eq. (2.2) [28]

$$j_1 = -1, \quad j_2 = 6, \quad j_3 = \frac{13}{2} + \frac{i\sqrt{71}}{2}, \quad j_4 = \frac{13}{2} - \frac{i\sqrt{71}}{2}, \tag{2.6}$$

where $i^2 = -1$. We see that two Fuchs indices are complex and Eq. (2.2) does not pass the Painlevé test. The Cauchy problem for Eq. (1.1) cannot be solved by the inverse scattering transform.

We obtain the following Laurent series for solution of Eq. (2.2)

$$y \simeq \frac{120k^3}{(z-z_0)^3} + \frac{\omega}{k} + a_6 (z-z_0)^3 - \frac{a_6^2 (z-z_0)^9}{1248k^3} + \cdots \tag{2.7}$$

From the Laurent series (2.7) we obtain the rational solution of Eq. (2.2) at $a_6 = 0$. It takes the form

$$y = \frac{\omega}{k} + \frac{120k^3}{(z-z_0)^3}, \tag{2.8}$$

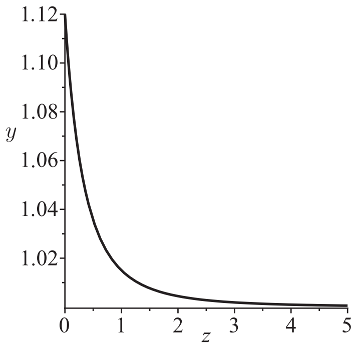

Fig. 1. Rational solution (2.8) of Eq. (2.2) at $k = 0.1; \omega = 0.1; z_0 = 1$.