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A note on using the Differential Transformation Method for the Integro-Differential Equations

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ABSTRACT

In this paper, we show that the generalization of the Differential Transformation Method (DTM) to Integro-Differential Equation is not the same thing that has been proposed in [P. Darania, A. Ebadian, A method for the numerical solution of the Integro-Differential Equation, Appl. Math. Comput. 188 (2007) 567–668]. We extend the integral part by the method of Jang et al. [2] for the Integro-Differential Equations. We give a counterexample for the method expressed by Darania and Ebadian [1] and some examples for comparing our modified method by method's of Darania and Ebadian [1].

1. Introduction

Consider linear Fredholm Integro-Differential Equation of the form

$$u'(x) + f(x)u(x) + \int_a^b k(x,t)u(t)dt = g(x), \tag{1.1}$$

$$u(a) = u_0, \tag{1.2}$$

where $f(x)$, $g(x)$ and $k(x,y)$ are sufficiently smooth real valued functions. The function $u(x)$ is unknown and a, b are constants. For $k(x,t) = 0$, Eq. (1.1) is converted to the ordinary differential equation

$$u'(x) = -f(x)u(x) + g(x), \tag{1.3}$$

with the initial condition (1.2). Details of DTM for ordinary differential equations may be found in [2]. The Taylor series expansion of $u(x)$ is defined as

$$u(x) = \sum_{i=0}^{\infty} \frac{u^{(i)}(a)}{i!}(x-a)^i. \tag{1.4}$$

If we denote $\frac{u^{(i)}(a)}{i!}$ by $U_a(i)$, then it follows from (1.4) that

$$u(x) = \sum_{i=0}^{\infty} U_a(i)(x-a)^i. \tag{1.5}$$

If this power series converges in $|x-a| < R$, then the derivative of this power series exists, is continuous (see [4]) and moreover

$$u'(x) = \sum_{i=0}^{\infty} (i+1)U_a(i+1)(x-a)^i. \tag{1.6}$$

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In DTM we also use the Taylor series expansion of $f(x)$ and $g(x)$ so we get

$$f(x)=\sum_{i=0}^{\infty}F_a(i)(x-a)^i, \tag{1.7}$$

$$g(x)=\sum_{i=0}^{\infty}G_a(i)(x-a)^i. \tag{1.8}$$

Substituting $f(x)$ and $g(x)$ from (1.7) and (1.8) in (1.1) and (1.2) we obtain a well known power series method that gives us a recurrence relation to find $U_a(i)$ and then we obtain $u(x)$ approximately. It is easy to show that

$$f(x)u(x)=\sum_{i=0}^{\infty}\sum_{l=0}^{i}F_a(i-l)U_a(l)(x-a)^i. \tag{1.9}$$

Substituting (1.5), (1.6), (1.8) and (1.9) in (1.3) and equating coefficients of $(x-a)^i$ we obtain the recurrence relation

$$(i+1)U_a(i+1)=-\sum_{l=0}^{i}F_a(i-l)U_a(l)+G_a(i)$$

and $u(a+h_1)$ is approximated by

$$u(a+h_1)=\sum_{i=0}^{m}U_a(i)h_1^i$$

as a new initial value at $a+h_1$, where $m\in\mathbb{N}$. We repeat this method by using the initial value $u(a+h_1)$ to obtain $u(a+h_1+h_2)$ in (1.3). Finally $u(a+h_1+\cdots+h_n)$ is obtained in a similar way.

2. Application of DTM for (1.1) and (1.2)

Now we extend this method for integral part as described in [3] and we use two-dimensional differential transform for the kernel of integral. By smoothness assumption of $k(x,t)$ we have

$$k(x,t)=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}K_a(i,j)(t-a)^j(x-a)^i, \tag{2.1}$$

where

$$K_a(i,j)=\frac{\partial^{(i+j)}}{i!j!\partial x^i\partial t^j}k(x,t)\bigg|_{x=a,t=a}.$$

It is also easy to show that

$$k(x,t)u(t)=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{j}K_a(i,j-l)U_a(l)(t-a)^j(x-a)^i. \tag{2.2}$$

By assuming that the right hand side of (2.2) is uniformly convergent in a neighborhood of (a,a) we can interchange this series with integral (see [4]) to obtain

$$\int_a^b k(x,t)u(t)dt=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{l=0}^{j}K_a(i,j-l)U_a(l)\int_a^b(t-a)^jdt(x-a)^i.$$

We now truncate the series to estimate the integral as

$$\int_a^b k(x,t)u(t)dt\simeq\sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{l=0}^{j}K_a(i,j-l)U_a(l)\int_a^b(t-a)^jdt(x-a)^i. \tag{2.3}$$

By substituting (1.5), (1.6), (1.8), (1.9) and (1.1) in (1.1) and equating coefficients of $(x-a)^i$ we obtain the following linear system for $i=0,\ldots,n-1$

$$(i+1)U_a(i+1)+\sum_{l=0}^{i}F_a(i-l)U_a(l)+\sum_{j=0}^{n}\sum_{l=0}^{j}K_a(i,j-l)U_a(l)\int_a^b(t-a)^jdt=G_a(i), \tag{2.4}$$

where $U(0)=u(a)$ and we have n unknown parameters ($U_a(i)$, for $i=1,\ldots,n$) with n equations. We solve this system by accepting $\sum_{i=0}^{n}U_a(i)h_1^i$ as an estimation of $u(a+h_1)$. After that we consider $u(a+h_1)=\sum_{i=0}^{n}U_a(i)h_1^i$ as the initial value of