Applied Mathematics and Computation 219 (2013) 7329–7337

Contents lists available at SciVerse ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Uniform stability of impulsive infinite delay differential equations with applications to systems with integral impulsive conditions ☆

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ARTICLE INFO

Keywords:
Uniform stability
Infinite delay differential equations
Integral impulsive conditions
Lyapunov–Razumikhin method
Impulsive perturbation
Impulsive control
Neural networks

ABSTRACT

In this paper, a class of impulsive infinite delay differential equations is considered. By employing Lyapunov–Razumikhin method and analysis techniques, several new sufficient conditions ensuring the uniform stability are obtained from impulsive perturbation and impulsive control point of view, respectively. The main advantage of those results is that they can be applied to the delay systems with integral impulsive conditions. As an application, we study a class of delayed neural networks with integral impulsive conditions and derive some results ensuring the uniform stability. Finally, two examples are given to show the effectiveness of the presented criteria.

1. Introduction

In the last couple of decades, the theory and application of impulsive functional differential equations (IFDEs) has undergone a rapid development. Many mathematical models in the study of neural networks, population dynamics, biology, physics and ecology, etc. can be modeled by IFDEs, see [1–11] and references therein. One of the main problems in the study of IFDEs is the stability problem of the solutions. Up to now, various stability concepts have been proposed and studied via different methods, such as Lyapunov functional or function methods connected with the Razumikhin technique [12–15], Comparison principle [16,17], fixed point theory [18] and inequality techniques [19,20], etc. For a detailed discussion on this topic we refer the reader to [21–27].

However, in connection with the work mentioned above, special attention is paid to the fact that much work above are characterized by the fact that the encountered instantaneous perturbations (i.e., impulsive effects) are only dependent on their current state at each moment of time. As we know, in certain circumstances a real system is usually affected by impulsive effects which in many cases are intermittent and that the processes under consideration depend on not only their current state but also the state in recent history. Thus, from a practical point of view, incorporating delay in the impulsive conditions ensures a better model of the process involved, see [28–34] and the references cited therein. For example, Akca et al. [29] investigated the global stability of additive Hopfield neural networks with integral impulsive conditions. In [30], Liu et al. studied the global exponential stability and exponential convergence rate for high-order delayed Hopfield neural

☆ This work was jointly supported by the Project of Shandong Province Higher Educational Science and Technology Program (No. J12LI04), the Research Fund for Excellent Young and Middle-aged Scientists of Shandong Province (No. BS2012DX039) and the National Natural Sciences Foundation of China: Tianyuan Foundation (No. 11226136 and 11171192).

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http://dx.doi.org/10.1016/j.amc.2012.12.033

networks with time-varying delays in impulsive conditions. In [32], Li et al. studied the existence, uniqueness and stability of recurrent neural networks with integral impulsive conditions and time delay in the leakage term. From the theoretical analysis point of view, however, so far there exists little work about the stability of IFDEs with delay in the impulsive conditions. Recently, by employing Lyapunov–Razumikhin method, Zhang and Sun [34] considered the stability of IFDEs with delay in the impulsive conditions. But those results can only be applied to IFDEs with discrete delay in the impulsive conditions and moreover they cannot be applied to IFDEs with infinite delays.

The purpose of this paper is to close the gap and establish some uniform stability criteria for IFDEs with infinite delays by employing Lyapunov–Razumikhin method and some analysis techniques. These results are given from impulsive perturbation and impulsive control point of view, respectively, and can be applied to finite or infinite delay systems with integral impulsive conditions. As an application, we consider a class of delayed neural networks with integral impulsive conditions and derive some results ensuring the uniform stability. The paper is organized as follows. In Section 2, we will provide some notations and definitions useful all along the paper. In Section 3, some new sufficient conditions on uniform stability of IFDEs are derived. Two examples are given to show the effectiveness of the presented criteria in Section 4. Section 5 concludes the paper.

2. Preliminaries

Notation 1. Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}_+$ the set of positive real numbers, $\mathbb{Z}_+$ the set of positive integers and $\mathbb{R}^n$ the n-dimensional real space equipped with the Euclidean norm $|\bullet|$. The impulse times t_k satisfy $0 \leqslant t_0 < t_1 < \ldots < t_k \to \infty$ as $k \to \infty$. $[\bullet]^*$ denotes the integer function. I denotes the identity matrix with appropriate dimensions and the notation $\star$ always denotes the symmetric block in one symmetric matrix. For any interval $J \subseteq \mathbb{R}$, set $S \subseteq \mathbb{R}^k (1 \leqslant k \leqslant n)$, $C(J,S) = \{\varphi : J \to S \text{ is continuous}\}$ and $PC(J,S) = \{\varphi : J \to S$ is continuous everywhere except at finite number of points t, at which $\varphi(t^+), \varphi(t^-)$ exist and $\varphi(t^+) = \varphi(t)\}$. Let $\mathbb{C}_\alpha$ be an open set in $PC([-\alpha,0],\mathbb{R}^n)$ and $\mathbb{C}_\tau$ be an open set in $PC([-\tau,0],\mathbb{R}^n)$, where $0 < \tau < \alpha \leqslant \infty$. $\mathbb{K} = \{a \in C(\mathbb{R}_+,\mathbb{R}_+) |\ a(0) = 0$ and $a(s) > 0$ for $s > 0$ and a is strictly increasing in $s\}$. For any $t \geqslant t_0 \geqslant 0 > -\alpha \geqslant -\infty$, let $f(t,x(s))$ where $s \in [t-\alpha,t]$ or $f(t,x(\cdot))$ be a Volterra type functional. In the case when $\alpha = +\infty$, the interval $[t-\alpha,t]$ is understood to be replaced by $(-\infty,t]$.

Consider the following IFDEs:

$$\begin{cases} x'(t) = f(t,x(\cdot)), & t \geqslant \sigma,\ t \neq t_k, \\ \Delta x|_{t=t_k} = x(t_k) - x(t_k^-) = I_k(t_k, x_{t_k^-}), & k \in \mathbb{Z}_+, \\ x(\sigma+s) = \phi(s), & -\alpha \leqslant s \leqslant 0, \end{cases} \tag{1}$$

where $\sigma \geqslant t_0 \geqslant 0, \phi \in \mathbb{C}_\alpha, f \in C([t_{k-1},t_k) \times \mathbb{C}_\alpha,\ \mathbb{R}^n)$. For each $t_k \geqslant t_0,\ x_{t_k^-} \in \mathbb{C}_\tau$ is defined by $x_{t_k^-}(s) = x(t_k^- + s),\ s \in [-\tau,0]$. For each $k \in \mathbb{Z}_+,\ I_k \in C([0,\infty) \times \mathbb{C}_\tau,\mathbb{R}^n)$. Define $PCB = \{\varphi \in \mathbb{C}_\alpha : \varphi$ is bounded$\}$ and $PCB_\delta = \{\varphi \in PCB : \|\varphi\| \leqslant \delta\}$ in which for $\varphi \in PCB$, the norm of φ is defined by $||\varphi|| = \sup_{-\alpha \leqslant \theta \leqslant 0}|\varphi(\theta)|$.

In this paper, we assume that functions $f, I_k, k \in \mathbb{Z}_+$, satisfy some necessary conditions for the global existence and uniqueness of the solutions for $t \geqslant t_0$, see [23,35,36] for detailed information. Denote by $x(t) = x(t,\sigma,\phi)$ the solution of (1) through (σ,ϕ). Moreover, we assume that $f(t,0) = 0,\ I_k(t_k,0) = 0,\ k \in \mathbb{Z}_+$, then $x(t) \equiv 0$ is a solution of Eq. (1), which is called the trivial solution.

Definition 2.1 [23]. The function $V : [\alpha,\infty) \times \mathbb{C}_\alpha \to \mathbb{R}_+$ belongs to class v_0 if

(1) V is continuous on each of the sets $[t_{k-1},t_k) \times \mathbb{C}_\alpha$ and $\lim_{(t,\varphi_1)\to(t_k^-,\varphi_2)} V(t,\varphi_1) = V(t_k^-,\varphi_2)$ exists;
(2) V (t,x) is locally Lipschitzian in x and $V(t,0) \equiv 0$.

Definition 2.2 [23]. Let $V \in v_0$, for any $(t,\psi) \in [t_{k-1},t_k) \times \mathbb{C}_\alpha$, the upper right-hand Dini derivative of V along the solution of Eq. (1) is defined by

$$D^+V(t,\psi(0)) = \limsup_{h\to 0^+} \frac{1}{h}\{V(t+h,\psi(0)+hf(t,\psi)) - V(t,\psi(0))\}.$$

Definition 2.3 [23]. The trivial solution of Eq. (1) is said to be

(P_1) stable, if for any $\sigma \geqslant t_0$ and $\varepsilon > 0$, there exists some $\delta = \delta(\varepsilon,\sigma) > 0$ such that $\phi \in PCB_\delta$ implies $|x(t,\sigma,\phi)| < \varepsilon,\ t \geqslant \sigma$;
(P_2) uniformly stable, if the δ in (P_1) is independent on σ.

3. Main results

Theorem 3.1. *Assume that there exist functions $w_1, w_2 \in \mathbb{K}, D_k \in PC(\mathbb{R}_+,\mathbb{R}_+), V(t,x) \in v_0$ and constants $C_k \geqslant 0,\ k \in \mathbb{Z}_+$, such that*