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Exploring collision-free path planning by using homotopy continuation methods

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ABSTRACT

Autonomous and semi-autonomous robots play significant roles in space and terrestrial exploration, even more in unfavorable and dangerous environments. Although recent advances allow robots to evolve in many such environments, one of the most important problems remains the establishment of collision-free trajectories in static or partially (temporal) static environments. This paper presents a different approach to address this problem, proposing a methodology based on homotopy continuation methods (HCM) capable of generating collision-free trajectories in two and three dimensions. The basic idea behind the proposal relies on the construction of a nonlinear equation representing the map of the environment, making it possible to apply HCM methods to obtain collision-free paths. A series of simulations are presented to show the effectiveness of the method avoiding circular, semi-rectangular, spherical shaped and semi-parallelepipeds obstacles.
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1. Introduction

Autonomous and semi-autonomous robots are important in fields as diverse as: industrial, military or medical, and in numerous applications as: terrestrial and space exploration, nuclear facilities inspection and, in general, for any unsafe or hostile environments. To execute many of the given tasks robots need sophisticated and highly accurate modules, particularly for perception and navigation [1]. To go from one point to another, generally a navigation module requires: (a) a map of the environment and (b) a path-planning algorithm.

Maps can be given to the robot as *a priori* information (e.g. a CAD model of the environment) or can be constructed by the robot itself in an exploration phase by a proper SLAM algorithm (Simultaneous Localization and Mapping) [2]. For the purposes of this work, is considered that the robot has already a map of the environment allowing us to focus on path planning algorithms.

Path-planning algorithms provide collision-free trajectories; however as they are very time consuming methods it is common to consider the environment as static. While for industrial environments or planetary exploration this statement is generally true, it is not the case for environments like: homes, offices and, in general, for most of the real-world environments [3–20]. Even so, it is possible to consider a temporal window where the environment could be considered as static.

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This window depends on environments and applications. Additionally, in this article we will only explore the collision-free trajectory planning, leaving as future work the problem of path following involving robot holonomic restrictions.

Nowadays, there are many methods for planning and generating collision-free trajectories. Roughly, we can group them into two classifications [21]: graphs and artificial potential fields. On one hand, the graphs approach solves the problem representing the collision free space by an interconnected graph. Then, a search is performed leading to a collision free trajectory, between initial and destination points. Nonetheless, the complexity of this kind of methods grows exponentially when the number of obstacles and possible solutions increase [22]. On the other hand, artificial potential fields methods work based on a simulation of repulsive forces around obstacles and an attractor located at the goal point. The solution to the potential field map is a collision-free trajectory. Unfortunately, solutions for these methods can be trapped by local minimums of the artificial field, providing wrong trajectories [23].

The use of homotopy as support technique for path planning was exposed in [24], showing a technique for trajectory replanning in environments with unpredictable changes. In the mentioned work, it is employed a homotopy principle for creating a set of homotopy paths from the original path. In addition, the valid homotopy path for the robot is selected using a reactive control algorithm. In [24–31] it is shown that homotopy continuation methods (HCM) can be useful for kinematics problems. Therefore, in this work is proposed the use of HCM methods to generate a family of collision-free homotopy paths in two or three dimensions.

First, we propose a methodology to establish a nonlinear algebraic equation that represents the map of obstacles, using as variables the relative displacement of the robot with respect to initial position. This nonlinear equation has a solution, exact at the goal point of the robot. Second, we construct a homotopy map that use, as starting point, the initial position of the robot. Finally, a numerical continuation procedure is employed as a mechanism to trace the collision-free path that ends at the goal point (solution).

This paper is organized as follows. In Section 2, we briefly review the basic idea of HCM methods. The homotopy path planning method (HPPM) is given in Section 3. The proposal of a modified version of the HPPM method is provided in Section 4. Section 5, refers to an extension of HPPM and EHPPM methods to three dimensions. Some numerical simulations are presented in Section 6. In Section 7, we summarize our results and present future research. Finally, a brief conclusion is given in Section 8.

2. Basis of homotopy continuation methods

Nonlinear algebraic equations systems (NAES) are very difficult to solve by conventional methods like Newton–Raphson (NR). The NR method converges properly if an initial guess for the solution is available, though there are examples for which the NR method diverges from all initial guesses. Nevertheless, it is common that this initial guess is not available or not simple to obtain. Therefore, convergence of the plain NR method cannot be guaranteed for all practical problems. Hence, one way to circumvent this issue is using HCM methods.

The HCM methods are a continuous transformation from one trivial problem (simple to solve) to the study problem (hard to solve). These kind of methods are applied to such diverse problems like: electronic circuits [32–43], Toeplitz systems, nonlinear control synthesis [44], stochastic finance economies [45], load flow solutions of ill-conditioned power systems [46], discretization of ordinary differential equations [47], inverse kinematics problems [24–31], optimization [48,49], among many others.

The first step to formulate a homotopy [50,33,35,51–53,34,54,48,55,28,49,56–60,36,32,61] is to establish a nonlinear equation that models the problem to be solved; which is defined as

$$f(x) = 0, \quad f :\in \Re^n \to \Re^n, \tag{1}$$

where x denotes the variables of the problem and n is the total number of those variables.

A homotopy map can be represented as:

$$H(f(x), \lambda) = 0, \quad H :\in \Re^{n+1} \to \Re^n, \tag{2}$$

where λ is the homotopy parameter.

Eq. (2) represents any homotopy formulation that fulfils the following conditions:

- For $\lambda = 0$, solution for $H^{-1}(0)$ is known or easily found using numerical methods.
- For $\lambda = 1$, $H(f(x), 1) = f(x)$. It means that at $\lambda = 1$ the solution or solutions for $f(x)$ can be found.
- The path for $H^{-1}(0)$ is a continuous function of λ with $0 \leqslant \lambda \leqslant 1$.

The homotopy path γ is the solution set for $H^{-1}(0)$, representing a continuous curve that can be traced by numerical continuation techniques or path following methods [62,51,63–65,37,38,29,30]. A possible homotopy map is

$$H(f(x), \lambda) = \lambda f(x) + (1 - \lambda) g(x) = 0, \tag{3}$$

where $g(x)$ is a problem easy to solve.

The selection $g(x)$ determines different specific homotopies. For instance, the Newton homotopy [38,55,66] can be obtained if we choose $g(x) = f(x) - f(x_0)$; after some algebraic steps results

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