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Numerical solution of stochastic partial differential difference equation arising in reliability engineering

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ABSTRACT

In this paper, a numerical method is proposed to solve the transient state of Markovian system of equations. Such equations appear in the field of reliability engineering for systems having variable failure and repair rates. Generally, steady state behaviour of the system model is studied due to some constraint on obtaining transient state solution. The proposed method helps to determine the probability values, by utilizing finite difference scheme iteratively in conjunction with the results of integral appearing in stochastic differential difference equation obtained using supplementary variable technique. This method also uses the Lagrange's method to interpolate the missing value of repair rates of the system wherever required in computation. Results thus obtained are found to be efficient for studying the transient state behaviour of the system.

1. Introduction

Since 1930 [1], many mathematical models have been discussed [2–4] which helped the manufacturing company to improve their manufactured product so that reliable product can go to the customers. Mathematical models like Fault Tree Analysis (FTA) [4], Failure Mode, Effects and Criticality Analysis (FMECA) [4], and Markov models are among the improvement techniques of the system performance. Markovian models are considered to be the most comprehensive technique being used today [5].

Performance analysis of systems using Markov models depends on transient as well as steady state solutions of the resulting mathematical equations. Markovian models are interpreted on the basis of steady state and transient analysis. Steady-state analysis has been the focus of most of the performance studies in the area of manufacturing systems. Though, the transient analysis plays an important role in studying issues like accumulated performance rewards over finite intervals, first passage times, sensitivity analysis, settling time computation, and deriving the behavior of queuing models as they approach equilibrium [6]. Transient solutions are important in reliability theory as they help to study the time dependent availability and reliability of the process.

Various techniques have been discussed [7–17] for evaluating the transient solution for the Markovian models having constant failure and repair rates whereas transient analysis for problems as discussed in [18–34] are yet to be tackled. In the present work, we are proposing a numerical method to solve stochastic partial differential difference equations of transient state, which occurred in reliability engineering while studying the performance of system. These equations are obtained by using supplementary variable technique [34] under variable failure and repair rates. The model considered in this paper has been taken from the works of Gaver [18]. (For more details on basic background see [1–3,6,35] and on advanced supplementary variable technique refer [19–34]).

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The paper is organized as follows: Section 2 deals with problem definition, where we discuss some of the existing research work related to problem stated. In Section 3, we have introduced the proposed numerical method to obtain the time dependent probabilities of various states of the system possessing variable failure and repair rates. The numerical solutions obtained from the proposed method have finally being analysed in Section 4. Certain conclusions based on the present study have also been discussed in this Section.

2. Problem definition

In reliability theory, Markov Chains are used to study the performance of system by defining the stochastic process under some stated assumption for the system under study. The performance of system is then analysed by assessing their state probabilities. However, a system having variable repair rates failed to possess Markov property [18,34]. As, it is very difficult to find the solution of non Markovian system, so such system is converted into Markovian in nature by introducing a supplementary variable technique as discussed in Cox [34] and Medhi [35].

For system performance, several authors [6–17] have obtained the transient solution of Markovian models by assuming constant repair and failure. Narahari and Viswanadham [6] discussed Lapalace transform inversion and matrix exponential methods to compute the transient solution of Markovian models by taking into account the methodology of [7,8]. They found difficulties while computation for large Markovian systems. Tombuyses and Devooght [9] investigated four explicit and six implicit Runge–Kutta methods for obtaining transient solution of such models emphasing on stability, amount of numerical work and accuracy to solve. Lindermann et al. [10] discussed numerical methods for reliability of Markov closed fault – tolerant system. Reliability and availability analysis of some system with common – cause failure was investigated by Abuelmaatti and Quamber [11] using Simulation Program with Integrated Circuit Emphasis (SPICE) circuit simulation program. Malhotra [12] discussed hybrid approach for transient solution of stiff and non-stiff Markov models. Recently, Amiri and Tari [13] employed the concept of eigenvalues and eigenvectors for analyzing the transient availability and survivability of the system. (See Refs. [14–17,20,29] for other related studies).

However, transient solution for Markovian models under variable repair rates are not yet obtained. But, such kind of Markov models under variable failure and repair rates are discussed by Gaver [18], Arora [19] and Sridharan and Mohanavadivu [20] for performance analysis of various systems using steady state only. Recently, several authors [21–28] have also analyzed the steady states for various manufacturing plants. In their studies, these authors developed complex mathematical equations consisting of simultaneous linear ordinary and partial differential equations which determine the reliability and availability of their respective industrial systems. They used Laplace Transform [19,21,24,26,31], Lagrange's methods [22,23,25,27,28] and separation of variable method [30] to solve the resultant system of simultaneous linear partial differential difference equations in their respective work. The analytical solutions used are so intricate that the industry persons may not use these results conveniently in the performance analysis of the system. Thus, there is need to apply some numerical methods which can be used to solve such kind of industrial problem. In this paper, an efficient numerical method is proposed to fill the gap. The proposed numerical method is combination of different numerical approaches. It consists of three numerical methods (Finite Difference schemes, Simpson's one-third method and Lagrange interpolation method) which iteratively compute the transient solution of the Markovian model taken under study and is named as Lagrange Finite Difference Simpson Method (LFDSM). This is simple approach and helps to approximate the solution of Markovian models having variable repair and failure rates.

In the present work, an attempt has been made to obtain a transient numerical solution for model equations discussed in Gaver [18]. The model considered in this paper is the simplest one in comparison to complex models available in the literature [19–33]. For studying the performance of system Gaver [18] discussed the following linear partial differential equations assuming variable repair rate and constant failure rate of the subsystems.

$$P_0'(t) = -\alpha P_0(t) + \int_0^x \beta(x) P_1(x,t)dx, \tag{1}$$

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial t}\right) P_1(x,t) + \beta(x) P_1(x,t) = \alpha P_0(t), \tag{2}$$

also,

$$P_1(t) = \int_0^\infty P_1(x,t)dx. \tag{3}$$

This system is subjected to the boundary condition

$$P_1(0,t) = -\alpha P_0(t), \tag{4}$$

and initial conditions

$$P_0(0) = 1, \quad P_1(x,0) = 0, \tag{5}$$