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# Reactive rimming flow of non-Newtonian fluids

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ABSTRACT

The steady and non-steady flows of a liquid polymer treated as a non-Newtonian fluid on the inner surface of a horizontal rotating cylinder are investigated. Since the Reynolds number is small and the liquid film is thin, a simple lubrication approximation is applied. Governing equations for non-steady Power-Law and Ellis fluids are solved numerically and the time of transition from non-steady to steady-state mode for various model parameters and flow conditions are defined. The stabilization effect of a chemical reaction within the polymeric fluid (reactive flow) is examined.



## 1. Introduction

The problem of rotational flow on the inner and/or outer wall of a hollow horizontal cylinder has been of interest for many years due to its wide range of applications in industry [1,2]. Moffatt [3] was the first to derive the condition for the maximal supportable load for a Newtonian liquid. Preziosi and Joseph [4] later presented the same result in another form and named it a run-off condition for coating and rimming flows.

The possible instability of the liquid film on a cylindrical surface is one the most challenging aspects of this problem. The highly unstable nature of rimming Newtonian flow was discussed in a number of publications [5–11]. Although the aforementioned investigations highlight the main characteristics of rimming flow, not enough has been done to show the effect of non-Newtonian properties on such flow, given its importance. Only a few attempts have been made. The power-law [12], Carreau–Yasuda [13], Ellis [14,15], Bingham [16], and viscoelastic models [17,18] have been used to study rimming flow. Fomin et al. [13,14] and Fomin [15] recently extended the estimates, originally made by Moffatt [3] for Newtonian fluids, for a generalized Newtonian fluid. Most polymeric solutes used in rotational coating are non-Newtonian liquids, which exhibit shear-thinning behavior for moderate to high shear rates. Liquid polymers behave as Newtonian liquids near the free surface (at very low shear rates) and exhibit non-Newtonian characteristics above a certain transitional shear rate, $\dot{\gamma}_t$. The importance of non-Newtonian effects for generalized Newtonian fluids is characterized by the shear-thinning number $Wi = \lambda\Omega/\delta$, where $\Omega$ is the characteristic angular velocity of the rotating cylinder, $\delta$ is the ratio of the characteristic thickness of the film and the radius of the cylinder, and $\lambda \approx (\dot{\gamma}_t)^{-1}$ is a typical time scale for liquid polymers, which is well documented [19] and normally stays in the interval of $(10^{-2}, 10^{-1})$ seconds. In some situations, e.g. a higher speed of rotation, a thinner liquid layer, or a smaller transition shear rate, the value of $Wi$ can be quite large. This illustrates the dominating role of non-Newtonian effects. Results of numerical computations available in [13] show that a Carreau fluid exhibits power-law behavior for large values of $Wi$. For instance, with $Wi = 8$, the results obtained with the Carreau and power-law models practically

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**Nomenclature**

| | |
|---|---|
| $c$ | concentration of reacting solute, $c = c/c_0$ |
| $c_0$ | initial concentration of reacting solute |
| $C_B$ | inverse of the Bond number |
| $De$ | Deborah number |
| $G$ | inverse function to $\eta(x)x$ |
| $g$ | gravitational acceleration |
| $h$ | thickness of the liquid layer, $h = h/h_0$ |
| $h_c$ | critical thickness of the liquid layer at $\theta = 0$ |
| $h_0$ | characteristic thickness of the liquid layer |
| $k$ | consistency parameter in a power-law constitutive equation |
| $n$ | flow index |
| $p$ | non-dimensional pressure, $p = p^*/(g\rho r_0)$ |
| $q$ | non-dimensional mass flux |
| $r$ | non-dimensional radial coordinate, $r = r^*/r_0$ |
| $r_o$ | radius of the cylinder |
| $R$ | non-dimensional radial coordinate, $R = (1 - r)/\delta$ |
| $Re$ | Reynolds number |
| $t$ | non-dimensional time, $t = t^*\Omega$. |
| $v_R, v_\theta$ | non-dimensional radial and azimuthal components of the fluid velocity, $v_\theta = v^*_\theta/(\Omega r_0)$, $v_R = v^*_r/(\Omega_0 r_0 \delta)$ |
| $Wi$ | shear-thinning number, $Wi = \lambda\Omega/\delta$ |

*Greek symbols*

| | |
|---|---|
| $\delta$ | ratio of the characteristic liquid layer thickness to the radius of the cylinder, $\delta = h_0/r_0$ |
| $\dot\gamma$ | shear rate |
| $\lambda$ | relaxation time |
| $\kappa$ | parameter in a kinetic equation that defines the rate of polymeric reaction |
| $\mu$ | function of deformation rate |
| $\eta$ | time-dependent viscosity |
| $\eta_0$ | characteristic viscosity ($\eta_0 = k(\Omega/\delta)^{n-1}$ for the power-law fluid) |
| $\theta$ | azimuthal coordinate |
| $\rho$ | liquid density |
| $\sigma$ | surface tension |
| $\tau_{\theta R}$, $\tau_{RR}$, $\tau_{\theta\theta}$ | non-dimensional components of the stress tensor deviator, $\tau_{R\theta} = \tau^*_{r\theta}/(\eta_0\Omega/\delta)$, $\tau_{RR} = \tau^*_{rr}/(\eta_0\Omega)$, $\tau_{\theta\theta} = \tau^*_{\theta\theta}/(\eta_0\Omega)$. |
| $u(t)$ | angular velocity of the cylinder |
| $\Omega$ | characteristic (maximal) angular velocity of the cylinder |

*Superscripts*

| | |
|---|---|
| $*$ | dimensional quantities |

*Subscripts*

| | |
|---|---|
| $0$ | characteristic quantity |
| $\theta$ | azimuthal component |
| $R$ | radial component |

coincide. A similar conclusion was made in [14] for Ellis fluids. Hence, flows that can be characterized by high values of the shear-thinning number can be modeled by the power-law constitutive equation.

Our main concern is the rotational molding of highly viscous polymers [1,2] that exhibit non-Newtonian shear thinning or shear thickening behavior. We are particularly interested in eliminating possible instabilities and providing the criteria for steady-state flow to obtain a continuous and smoothly coated film on the wall of the horizontal cylinder. Our research focuses on the analysis of the rimming flow of highly viscous non-Newtonian polymeric fluids and the transition of this time-dependent flow to a steady-state mode. Analysis of non-steady non-Newtonian rimming flow is based on the numerical solution of the governing equation of the liquid film thickness for power-law and Ellis fluids. The factors that affect the length of the transition period from non-steady to steady state mode are defined, and the behavior of the liquid film during this period is illustrated. For reactive rimming flow, the stabilizing effect of the viscosity-increasing chemical reaction is demonstrated.