Applied Mathematics and Computation 219 (2013) 4465–4473

Contents lists available at SciVerse ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Absolute stability of Lurie direct control systems with time-varying coefficients and multiple nonlinearities

Di Wang [a,b], Fucheng Liao [a,*]

[a] School of Mathematics and Physics, University of Science and Technology Beijing, Beijing, China
[b] School of Automation, University of Science and Technology Beijing, Beijing, China

ARTICLE INFO

Keywords:
Lurie systems with time-varying coefficients
Absolute stability
Lyapunov method
Nonsingular M-matrix
Upper limit
Diagonally dominant matrix

ABSTRACT

Absolute stability of Lurie direct control systems with time-varying coefficients and multiple nonlinearities is studied in this paper. Depending on the related methods, relative magnitude of the norm-unbounded coefficients was estimated. By knowledge of nonsingular M-matrix, the Lyapunov function was constructed, and some absolute stability criteria of this kind of systems were obtained. In addition, some simple and practical corollaries were derived from the theorem of Taussky. The main contribution of this paper is that the criteria which we introduced allow for the situation that the norms of coefficient matrices are unbounded. At last, the example shows the availability of the criteria.

1. Introduction

Lurie control system is an important kind of nonlinear systems, and the absolute stability of this system has been extensively studied [1–3]. In [4], absolute stability of a class of Lurie indirect control large-scale systems was studied. Chen et al. [5] promoted the results of [4] and studied parametric absolute stability of interconnected Lurie systems. In [6], the absolute stability of Lurie system with multiple time-delays and nonlinearities has been considered, based on the approaches of decomposing the matrices and adding modulatory matrices, a novel delay-dependent sufficient condition for the absolute stability was proposed. Nam and Pathirana [7] studied absolute stability of Lurie control systems with multiple time-delays, by using extended Lyapunov functional, they derived improved stability criteria, which were strictly less conservative. In [8], the Lurie nonlinear systems with time-varying delay was considered, by employing an augmented Lyapunov–Krasovskii functional and retaining some useful terms which were used to be ignored in the derivative, new free-weighting matrix approach to represent the relationship among the time-varying delay has been introduced. As a result, some less conservative delay-dependent stability conditions were obtained in terms of linear matrix inequalities (LMIs). By using a new Lyapunov–Krasovskii functional, which splits the whole delay interval into two subintervals, some new delay-dependent robust absolute stability criteria of Lurie systems with time-varying delay and norm-bounded parameter uncertainties were obtained in [9].

However, the researches of Lurie control systems mentioned above are still in constant coefficients or norm-bounded coefficients, the absolute stability of Lurie control system with norm-unbounded coefficients is still not involved. Liao et al. [10,11] discussed the absolute stability of Lurie indirect control systems with norm-unbounded coefficients by an ingenious method, some criteria of this kind of systems were obtained. Nevertheless, owing to the dependencies between variables [1], the difficulty of the study in direct systems was increased, and there have no criterion to determine the absolute stability of the Lurie direct control systems with norm-unbounded coefficients until now. This paper will study the absolute

* Corresponding author.
E-mail address: fcliao@ustb.edu.cn (F. Liao).

0096-3003/$ - see front matter
http://dx.doi.org/10.1016/j.amc.2012.10.050

stability of Lurie direct control systems with time-varying coefficients and multiple nonlinearities, the criteria, introduced in this paper, can not only be used in the direct Lurie control systems with norm-unbounded coefficients and multiple nonlinearities, but also can be used in the direct Lurie control systems with norm-bounded coefficients.

Notation: Throughout this paper, $\lambda(A)$ represents the eigenvalue of the Matrix A ; Let vector $x = (x_1\, x_2\, \ldots\, x_m)^T$, and $y = (y_1\, y_2\, \ldots\, y_m)^T$, $x \leqslant y$ means that $x_i \leqslant y_i (i = 1, 2, \ldots, m)$; $||x||$ represents the Euclid norm of a vector x , that is $\|x\| = \sqrt{\sum_{i=1}^{n} x_i^2}$; $||A||$ represents the Euclid norm of a matrix A which induced by the Euclid vector norm $||x||$, i.e. $\|A\| = \max_{\|x\|=1} \|Ax\|$, and it can be easily proved that $\|A\| = \sqrt{\lambda_{\max}(A^T A)}$.

2. Problem statement

Consider the following Lurie control system with time-varying coefficients and multiple nonlinearities:

$$\begin{cases} \dot{x} = A(t)x + \sum_{j=1}^{m} b_j(t) f_j(\sigma_j), \\ \sigma_i = c_i^T(t)x, \quad (i = 1, 2, \ldots, m), \end{cases} \tag{1}$$

where $x \in R^n$ is the state; $b_i(t), c_i(t) \in R^n (i = 1,2,\ldots,m)$ are vector function, $b_i(t)$ continuous and $c_i(t)$ have first-order derivative in $(\tau,+\infty)$, there $\tau \in R$ or $\tau = -\infty$; $A(t)$ is an $n \times n$ matrix function, and continuous in $(\tau,+\infty)$. The nonlinearity $f_i(\cdot)(i = 1,2,\ldots,m)$ are continuous function, and they satisfy:

$$f_i(\cdot) \in K[0, +\infty] = \{f_i(\cdot) | f_i(0) = 0, 0 < \sigma f_i(\sigma) < +\infty, \sigma \neq 0\}.$$

The system (1) is said to be absolutely stable if its zero solution is globally asymptotically stable for any nonlinearity $f_i(\sigma_i) \in K[0,+\infty)$ [12].

3. Main results

The following basic assumptions are needed for our main results.

A1: We assume that there exists T satisfies $T \in (\tau,+\infty)$, and for any $t > T$, there exists positive definite symmetrical constant matrices P such that

$$\lambda(A^T(t)P + PA(t)) \leqslant -\delta(t) \leqslant -\delta,$$

where $\delta > 0$ is constant.

Remark 1. The condition A1 can ensure the global asymptotically stability of system $\dot{x} = A(t)x$ according to the Krasovskii theorem [13].

A2: For any $t > T$, we assume that

$$c_i^T(t) b_i(t) \leqslant -\theta_i(t), \quad (i = 1, 2, \ldots, m),$$

where $\theta_i(t) > 0$ $(i = 1,2,\ldots,m)$ are a known function.

Remark 2. In [12], we know that $c_i^T b_i < 0$ is necessary conditions of the absolute stability in the case of Lurie direct control systems with constant coefficient. So the condition A2 is exceedingly weak, but it is the essence of this paper.

A3: For any $t > T$, we assume that

$$\frac{2\|Pb_i(t)\|}{\sqrt{\delta(t)\theta_i(t)}} \leqslant \zeta_i, \frac{\|c_i^T(t)A(t) + \dot{c}_i^T(t)\|}{\sqrt{\delta(t)\theta_i(t)}} \leqslant \eta_i, \quad (i = 1, 2, \ldots, m),$$

where ζ_i, η_i $(i = 1,2,\ldots,m)$ are constant.

A4: For any $t > T$, we assume that

$$\frac{|c_i^T(t) b_j(t)|}{\sqrt{\theta_i(t)\theta_j(t)}} \leqslant \mu_{ij}, (i,j = 1, 2, \ldots, m; i \neq j),$$

where μ_{ij} $(i,j = 1,2,\ldots,m; i \neq j)$ are constant.

As we all known that the norm-unbounded and time-varying coefficients in system (1) are difficult to deal with, but using the $\delta(t)$, $\theta_i(t)$ in conditions A1, A2, and placing them in denominator, the relative magnitude of the norm-unbounded coefficients in system (1) can be estimate in conditions A3, A4. That is the "unlimited" nature can be expressed by the "limited" form, which makes feasibility for the study of absolute stability of system (1). Therefore, we have the following results.