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Induction heating of thin metal plates in time-varying external magnetic field solved as nonlinear hard-coupled problem

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ABSTRACT

A novel mathematical model of local induction heating of very thin metal plates in an external time-variable magnetic field is suggested. Distribution of eddy currents induced in the plate is modelled by electric vector $\boldsymbol{T}$-potential and thermal fluxes in the systems are expressed in terms of heat sources and sinks. Analysed are also the thermoelastic displacements in the plate, their back influence on the discretization mesh and distribution of the electromagnetic and thermal quantities. Numerical solution of the problem is carried out in the hard-coupled formulation. The methodology is illustrated by a typical example whose results are discussed.

1. Introduction

Local induction heating of thin metal plates is often used before their hot forming, annealing, and some other technological processes. From the physical viewpoint, it represents a relatively complicated nonlinear and nonstationary coupled problem, characterised by interaction of three fields: the primary electromagnetic field producing the temperature field and consequently the field of thermoelastic displacements. These displacements may generally change the geometry of the plate and, therefore, the original distributions of the electromagnetic and thermal quantities.

Modelling of this process is still a challenging business and the authors did not find any paper that would describe a complete solution of the above task. Of course, induction heating itself was modelled many times and relevant analytical and numerical methods abound. The most frequent approach is based on the determination of the magnetic vector potential $\boldsymbol{A}$ in the system (see, for example [1–4]), and this approach is also implemented in most of the available commercial codes (ANSYS, FLUX, OPERA, etc.). Its application, however, can fail when the arrangement solved is characterised by geometrically incommensurable dimensions of some structural elements (which means, that one dimension of such an element is much smaller than the other dimensions, but yet, this dimension is also important and its influence cannot be neglected). Such cases are typical by severe complications in the course of meshing and numerical solution itself.

We can mention, for example, local induction heating of nonferromagnetic thin plates or thin-wall pipes. One of the possible arrangements is depicted in Fig. 1. A nonferromagnetic plate **1** of a very small thickness $\delta \ll a, b$ (This thickness must also be substantially smaller than the depth of penetration.) is locally heated by eddy currents $\boldsymbol{J}_{\mathrm{ind}}$ generated by time varying magnetic field $\boldsymbol{B}_{\mathrm{ext}}$ produced in magnetic circuit **2** by field coil **3** and concentrated by appropriately shaped magnetic focusators **4.1** and **4.2**.

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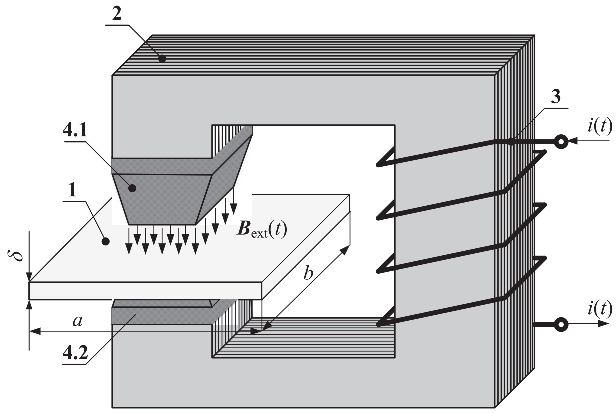

Fig. 1. Local induction heating of a thin plate: **1** – thin nonferromagnetic plate, **2** – laminated magnetic circuit, **3** – field coil, **4.1** and **4.2** – ferromagnetic focusators.

Handling the problem as a geometrically 3D task (depending on quantities x, y, z, t) formulated in the classical manner (in terms of the magnetic vector potential $\boldsymbol{A}$), and solving it by the finite element method is often unreal. This is because the thickness δ of the plate is negligible with respect to its remaining dimensions, which represents the fundamental complication for building the finite-element mesh.

On the other hand, considering the problem as a 2D task (thickness δ being neglected) is also counterproductive because in this case it is not possible to numerically approximate the boundary conditions for the magnetic vector potential $\boldsymbol{A}$. For solving problems of that kind the authors suggest an alternative method that works with the electric vector potential $\boldsymbol{T}$. Another improvement of the model consists in using a modified heat-transfer equation including heat sources and sinks that replace the boundary conditions of the temperature field.

2. Formulation of the problem

Consider a thin, nonferromagnetic, electrically conductive plate of any shape, whose surface is denoted by symbol Ω_1, its boundary being Γ, whose thickness $\delta \to 0$ and electrical conductivity is γ_{el}. The plate is locally (in an area $\Omega_2 \subset \Omega_1$) exposed by a time variable external magnetic field $\boldsymbol{B}_{\mathrm{ext}}(t)$, whose time evolution is known (see Fig. 2).

The task is to map the time evolution of distributions of current densities induced in the plate, volumetric Joule losses, temperature, and thermoelastic displacements. Unlike [5] where the authors solved the task disregarding the back influence of the varying geometry of the plate, now this effect is included in both mathematical and computer models. The numerical solution is performed by our own code written in Free Pascal [6].

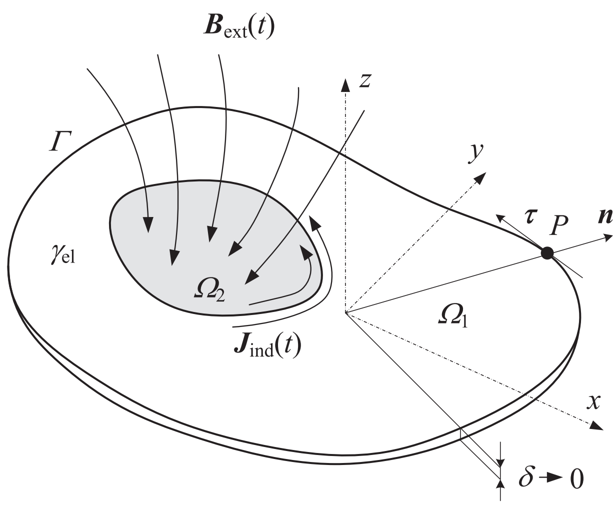

Fig. 2. Basic arrangement of the investigated system.