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Analysis of coupled transport phenomena in concrete at elevated temperatures

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ABSTRACT

In this paper, we study a non-linear numerical scheme arising from the implicit time discretization of the Bažant–Thonguthai model for hygro-thermal behavior of concrete at high temperatures. Existence of the time-discrete solution in two dimensions is established using the theory of pseudomonotone operators in Banach spaces. Next, the spatial discretization is accomplished by the conforming finite element method. An illustrative numerical example shows that the model reproduces well the rapid increase of pore pressure in wet concrete due to extreme heating. Such phenomenon is of particular interest for the safety assessment of concrete structures prone to thermally-induced spalling.

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1. Introduction

The hygro-thermal behavior of concrete plays a crucial role in the assessment of the reliability and lifetime of concrete structures. The heat and mass transfer processes become particularly important at high temperatures, where the increased pressure in pores may lead to catastrophic service failures. Since high-temperature experiments are very expensive, predictive modeling of humidity migration and pore pressure developments can result in significant economic savings. The first mathematical models of concrete exposed to temperatures exceeding 100 °C were formulated by Bažant and Thonguthai [1]. Since then, a considerable effort has been invested into detailed numerical simulations of concrete structures subject to high temperatures. However, much less attention has been given to the qualitative properties of the model, as well as of the related numerical methods.

In particular, the only related work the authors are aware of is due to Dalík et al. [3], who analyzed the numerical solution of the Kiessl model [8] for moisture and heat transfer in porous materials. They proved some existence and regularity results and suggested an efficient numerical approach to the solution of the resulting system of highly non-linear equations. However, the Kiessl model is valid for limited temperature range only and as such it is inappropriate for high-temperature applications. In this contribution, we extend the work [3] by proving the existence of an approximate solution for the Bažant–Thonguthai model, arising from the semi-implicit discretization in time. A fully discrete algorithm is then obtained by standard finite element discretization and its performance is illustrated for a model problem of a concrete segment exposed to transient heating according to the standard ISO fire curve. Here, the focus is on the short-term pore pressure build up, which is decisive for the assessment of so-called thermal spalling during fire.

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At this point, it is fair to mention that the Bažant–Thonguthai model was later extended towards more detailed multiphase description, see e.g. the works of Gawin et al. [6], Tenchev et al. [13] and Davie et al. [4] for specific examples. When compared to the original version, these advanced models provide better insight into physical and chemical processes in concrete (such as influence of gel water, pore water, capillary water, chemical reactions at elevated temperatures, etc.). Such potential increase in accuracy, however, comes at the expense of increased number of parameters, which typically reflect complex multi-scale nature of concrete. Hence, their experimental determination is rather complicated and the parameters can often only be calibrated by sub-scale simulations. Therefore, in this work we adopt a pragmatic approach and consider the single-phase Bažant–Thonguthai model with parameters provided by heuristic relations, obtained from regression of reliable macroscopic experiments.

The paper is organized as follows. In Section 2, we present the general single-phase, purely macroscopic, model for prediction of hygro-thermal behavior of heated concrete. In Section 3, we introduce basic notation, the appropriate function spaces and formulate the problem in the strong and variational sense. In Section 4.1, we specify our assumptions on data and structure conditions to obtain a reasonably simple but still realistic model of hygro-thermal behavior of concrete at high temperatures due to Bažant and Thonguthai [1]. An application of the Rothe method of discretization in time leads to a coupled system of semilinear steady-state equations, which (together with the appropriate boundary conditions) form a semilinear elliptic boundary value problem, formulated in the form of operator equation in appropriate function spaces. The existence result for this problem in space $W^{1,p}(\Omega)^2$ with $p \in (2,4)$ is proven in Section 4.2 using the general theory of coercive and pseudomonotone operators in Banach spaces. Next, the problem is resolved using the finite element method as presented in Section 5.1. In Section 5.2, numerical experiments are performed to investigate the moisture migration, temperature distribution and pore pressure build up in the model of concrete specimen exposed to fire, including a simple engineering approach to study the spalling phenomenon.

2. The coupled model for wet concrete

2.1. Conservation laws

The heat and mass transport in concrete is governed by the following system of conservation laws:

energy conservation equation:

$$\frac{\partial \mathscr{H}(\theta, w)}{\partial t} = -\nabla \cdot \boldsymbol{J}_\theta(\theta, w, \nabla\theta, \nabla w) + C_w \boldsymbol{J}_w(\theta, w, \nabla\theta, \nabla w) \cdot \nabla\theta; \tag{1}$$

water content conservation equation:

$$\frac{\partial \mathscr{M}(\theta, w)}{\partial t} = -\nabla \cdot \boldsymbol{J}_w(\theta, w, \nabla\theta, \nabla w). \tag{2}$$

The primary unknowns in the balance Eqs. (1) and (2) are the temperature θ and the water content w; w represents the mass of all evaporable water (free, i.e. not chemically bound) per m^3 of concrete. Further, $\mathscr{H}$ and $\mathscr{M}$ represent the amount of (internal) energy and the amount of free water, respectively, in 1 m^3 of concrete, $\boldsymbol{J}_\theta$ is the heat flux, C_w the isobaric heat capacity of bulk (liquid) water and $\boldsymbol{J}_w$ the humidity flux.

2.2. Constitutive relationships for heat and moisture flux

Following [1], the heat flux $\boldsymbol{J}_\theta$ arises due to the temperature gradient (Fourier's law) and due to the water content gradient (Dufour flux)

$$\boldsymbol{J}_\theta(\theta, w, \nabla\theta, \nabla w) = -D_{\theta w}(\theta, w)\nabla w - D_{\theta\theta}(\theta, w)\nabla\theta \tag{3}$$

and the flux of humidity $\boldsymbol{J}_w$ consists of the flux due to the humidity gradient (Fick's law) and due to the temperature gradient (Soret flux)

$$\boldsymbol{J}_w(\theta, w, \nabla\theta, \nabla w) = -D_{ww}(\theta, w)\nabla w - D_{w\theta}(\theta, w)\nabla\theta, \tag{4}$$

where $D_{\theta w},\ D_{\theta\theta},\ D_{ww}$ and $D_{w\theta}$ are continuous diffusion coefficient functions depending non-linearly on θ and w.

2.3. Boundary and initial conditions

To complete the introduction of the model, let us specify the boundary and initial conditions on θ and w. The humidity flux across the boundary is quantified by the Newton law:

$$\boldsymbol{J}_w(\theta, w, \nabla\theta, \nabla w) \cdot \boldsymbol{n} = \gamma_c(w - w_\infty), \tag{5}$$

where the right hand side represents the humidity dissipated into the surrounding medium with water content w_∞, specified in terms of the film coefficient γ_c. As for the heat flux, we shall distinguish the convective and radiation boundary conditions given by

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