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Sensitivity of optimal control for diffusion Hopfield neural network in the presence of perturbation ☆

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ABSTRACT

With the wide investigation of Hopfield neural network (HNN) and its control problems, this paper is to control diffusion HNN system in the presence of perturbation (disturbance, uncertainties) in the control field. In particular, it is try to answer the most interesting question on the sensitivity of these perturbations both in theoretical and computational aspects.

1. Preliminaries

A great deal contributed works are reported in the studying Hopfield neural networks (HNNs) respect to theoretically problems, see [2–6,8,7,13,15]. Even inclusive of our proposed diffusion HNN system in [12], the existing research is limited to the theoretic and computational issues, and lost of generality and realizability. How about working on real neural networks? Could our conclusion be simply applied to supposed HNN structure? What will be happened in the meaning of physical viewpoint? Keep these questions in our mind to consider control problem for perturbed HNN system. Actually, numerous factors of disturbances and uncertainties would be involved in our extracted neural network as mathematics model. Furthermore, these perturbations can significantly change the experimental results if proceeding of the investigation on neural networks consist of large-number of biological neurons. How about the sensitivity of each physics factor? All of these questions will attract our attention definitely.

The motivation of this paper is to directly manipulate the perturbed HNN with diffusion term. Hopefully, the current research results could provide us the access and direction in continuing investigation. Briefly to review our research papers as follows. Theoretic result reported at [12] in the framework of variational method, numerical solution of diffusion HNN had been handled in [17], 1D numerical approach are shown in [18,19], bang–bang control can be found in [16], pointwise control is published in [20]. Recent published paper [21] is focus on boundary pointwise control of diffusion HNN in two dimension case.

The objective of this work is aimed at controlling of diffusion HNN in the presence of disturbances and uncertainties. Meanwhile, it is quite desired to achieve the control in the realistic sense.

The article is organized by the following sections. Section 2 is to give perturbed HNN model with necessary notations and mathematical setting in the framework of variational method. Section 3 is to address control theory for such a diffusion HNN in the presence of perturbation arising in control field. Section 4 roughly state the adapted numerical computation algorithm

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based on the minimization of defined quadratic cost function. Section 5 is to show the demonstration results for three neurons for perturbed control field. The comparison with no perturbed case is also presented with simulated graphics for various physical quantities.

2. Perturbed HNN model

Diffusion term has been included in our physical model for having a nonlinear neural network model starting in [12]. For simplicity, this article is restricted for perturbation occurred in control field. As to a neuron system, the control might be language, electric current, drug, magnetic source, light source and so on. If these types of control variables are used to control HNN, perturbation or disturbance will be happen without doubt.

Let Ω be an open bounded domain of $\mathbf{R}^3$ with a piecewise smooth boundary $\Gamma = \partial\Omega$. Denote $\boldsymbol{x} = (x_1, x_2, x_3)$. Let $Q = (0,T) \times \Omega$ and $\Sigma = (0,T) \times \Gamma$ with $T > 0$. Let $y_{\mathbf{i}}(\boldsymbol{x}, t)$ denote the activation potential of the ith neuron for $i = 1, 2, \ldots, n$. The diffusion HNN model with perturbation is described by simultaneous system of n-numbers neurons

$$C_i \frac{\partial y_{\mathbf{i}}(t)}{\partial t} - d_{\mathbf{i}} \Delta y_{\mathbf{i}}(t) = -\frac{y_{\mathbf{i}}(t)}{R_{\mathbf{i}}} + \sum_{\mathrm{j}=1}^{n} F_{ij} f_{\mathbf{j}}(u_{\mathbf{j}}(t)) + (u_{\mathbf{i}}(t) + \delta u_{\mathbf{i}}) \quad \text{in } Q \tag{1}$$

with boundary condition $\frac{\partial y_{\mathbf{i}}(t)}{\partial \eta} = 0$ on Σ, and initial guess $y_{\mathbf{i}}(\boldsymbol{x}, 0) = y_0^i(\boldsymbol{x})$ for $i = 1, 2, \ldots, n$. Here in (1), $C_{\mathbf{i}}$ denote the total input capacitance of the amplifier ith and its associated input lead. $d_{\mathbf{i}} > 0$ are diffusion constants, F_{ij} are connection weight constants. The magnitude of $F_{ij} = \frac{1}{R_{ij}}$, where R_{ij} is the resistor connecting the output of j to the input line i, while the sign of F_{ij} is determined by the choice of the positive or negative output of amplifier jth at the connection site. $R_{\mathbf{i}}$ defined as $\frac{1}{R_{\mathbf{i}}} = \frac{1}{p_{\mathbf{i}}} + \frac{1}{R_{ij}}$, where p_i is the input resistance of amplifier ith. $f_j : \mathbf{R} = (-\infty, \infty) \to (-1, 1)$ are nonlinear sigmoidal activation functions, e.g., $f_j(s) = \tanh s$. Particularly, $u_i(t)$ denotes external current to i-neuron, i.e. control inputs, and δu_i represents perturbation term in control field.

In general, suppose δu_i is bounded for well-defined issue, that is, there exist constant C such that

$$|\delta u_i| \leqslant C, \quad \forall t \in [0, T].$$

Unbounded control input u_i is exclusive of our research for lost generality, and make non-sense for biological neural network. For virtual artificial neural networks, it needs to set the physical model, and deduce the corresponding theoretical and computational results.

Denote $\boldsymbol{u}(t) = (u_1(t), u_2(t), \ldots, u_n(t))$, and its the space $\mathcal{U} = L^2(0,T)^n$. Let $\mathcal{U}_{ad}$ be a closed and convex admissible set of $\mathcal{U}$. In order to find quantum optimal control pairing $\boldsymbol{u}^* = (u_1^*, u_2^*, \ldots, u_n^*)$ in system (1), define two Hilbert spaces $H = L^2(\Omega)$ and $V = H^1(\Omega)$ corresponding to Neumann boundary condition of systems (1). They are endowed with the usual inner products $(,),\ ((,))$ and norms $|\cdot|,\ \|\cdot\|$, respectively (cf. [1,11]). Then the embeddings in Gelfand triple spaces $V \hookrightarrow H \hookrightarrow V'$ are continuous, dense and compact. The associated cost function subject to system (1) is

$$J(\boldsymbol{u}) = \epsilon \sum_{i=1}^{n} \left\| y_i^f(\boldsymbol{u}) - y_i^{\text{target}} \right\|_V^2 + \sum_{i=1}^{n} (u_i, u_i)_{\mathcal{U}}, \quad \forall \boldsymbol{u} = (u_1, u_2, \ldots, u_n) \in \mathcal{U}, \tag{2}$$

where $y_i^{\text{target}} \in V, i = 1, 2, \ldots, n$ are target states, and $y_i^f(\boldsymbol{u}), i = 1, 2, \ldots, n$ are final observed states at time t_f, respectively. Here ϵ are weighted coefficients for balancing the evaluates of inherent and running costs.

Denote dual space of V by V', and the symbol $\langle\ ,\ \rangle$ denotes the dual pairing from V and V'. Denote $\psi = (\psi_1, \psi_2, \ldots, \psi_n)^t$ and $\phi = (\phi_1, \phi_2, \ldots, \phi_n)^t$, then the Hilbert spaces $\mathcal{H} = L^2(\Omega)^n$ and $\mathcal{V} = H^1(\Omega)^n$ with inner products defined by

$$(\psi, \phi)_{\mathcal{H}} = \sum_{i=1}^{n} (\psi_i, \phi_i), \quad \psi, \phi \in \mathcal{H},$$

$$(\psi, \phi)_{\mathcal{V}} = \sum_{i=1}^{n} (\psi_i, \phi_i), \quad \psi, \phi \in \mathcal{V},$$

respectively. Then dual space $\mathcal{V}' = (V')^n$ and dual pairing between $\mathcal{V}'$ and $\mathcal{V}$ is given by

$$\langle \psi, \phi \rangle_{\mathcal{V}, \mathcal{V}'} = \sum_{i=1}^{n} \langle \psi_i, \phi_i \rangle, \quad \psi \in \mathcal{V}, \phi \in \mathcal{V}',$$

where $\langle \psi_i, \phi_i \rangle$ denotes the dual pairing between V and V'. The norms of $\mathcal{H}$ and $\mathcal{V}$ are denoted by $|\psi|_{\mathcal{H}}$ and $\|\psi\|_{\mathcal{V}}$, respectively. Let us define the following vectors and matrix representations: