Applied Mathematics and Computation 219 (2012) 3923–3937

Contents lists available at SciVerse ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Delay can stabilize: Love affairs dynamics

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ARTICLE INFO

Keywords:
Time delay
Stability switches
Dyadic interactions
The Hopf bifurcation
Stabilization of steady state

ABSTRACT

We discuss two models of interpersonal interactions with delay. The first model is linear, and allows the presentation of a rigorous mathematical analysis of stability, while the second is nonlinear and a typical local stability analysis is thus performed. The linear model is a direct extension of the classic Strogatz model. On the other hand, as interpersonal relations are nonlinear dynamical processes, the nonlinear model should better reflect real interactions. Both models involve immediate reaction on partner's state and a correction of the reaction after some time.

The models we discuss belong to the class of two-variable systems with one delay for which appropriate delay stabilizes an unstable steady state. We formulate a theorem and prove that stabilization takes place in our case. We conclude that considerable (meaning large enough, but not too large) values of time delay involved in the model can stabilize love affairs dynamics.

1. Introduction

1.1. Delay in human emotions

In psychology of emotion, this formation of emotion is described by so-called differential emotions theory (DET) in which forming an emotion can be modeled by a dynamics of a system undergoing neurohormonal, motoric and experiential processes [1]. The core assumption in DET is that small set of emotions are primary and independent: joy, interest, sadness, fear, surprise and disgust [2]. These are independent because they achieve consciousness rapidly and automatically, and influence subsequent perception and cognition. The second important claim is that these emotions are discrete (associated with a specific neuromuscular pattern of facial movements) and distinguishable. Individual emotions also undergo interactions with other emotions in order to form emotion patterns that stabilize over repetitions and time and refer to compound emotions. Thus, single emotions are both the product and the subject of system organization. The systems are self-organizing in the sense that recursive interactions among component processes generate emergent properties.

There are two groups of compound emotions in terms of time necessary for the system to form them. First group is emotions that can be both close to immediate, or delayed: anxiety, anger, irritability, guilt, feeling overwhelmed, grief, hopelessness. However, there is also a group of emotions that are always delayed because it takes long for the system to reach them: feeling abandoned, resentment, feeling of alienation, withdrawal, numbness, depression.

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http://dx.doi.org/10.1016/j.amc.2012.10.028

1.2. Modeling of interpersonal relationships

Understanding the dynamics of marriage or other close personal relationships is a goal of various sociological and socio-psychological studies, including studies based on the mathematical sociology approach. Numerous papers presenting dynamical systems describing such relationships can be found, cf. e.g. [3,4,5,6,7,8,9,10,11,12,13,14,15,16,17]. Furthermore, in 1994 Gottman et al. published the text-book focused on that problem, where they used mathematical modeling in the description of different types of marriage and divorce prediction. Typically, the discrete dynamical systems approach is used in this type of modeling, focusing on nonlinear dynamics of interactions. However, continuous dynamical systems can be used as well, not excluding linear ODEs systems.

Models describing social interactions between agents using systems of ordinary differential equations (ODEs), especially derived from the game theory, are quite common nowadays, cf. the classic text-book of [18] or article by Axelrod [19] and the references therein. Also models describing human interactions as a process of responsive behavior has been considered by Platkowski and Poleszczuk [20]. In this section, we would like to discuss a problem of modeling personal interactions with delay, and provide argumentation for the delay to be incorporated into the modeling, while in Section 2 we introduce the model itself, and describe the meaning of variables to be used.

Models with time delays are quite often used to describe population dynamics, cf. e.g. a text-book by Gopalsamy [21]. It is obvious that time delays can be always observed in natural environment: it takes some time for the signal to travel from one cell to the other, the process of transcription of a protein also takes some time, cf. e.g. [22] or [23].

We find it reasonable and necessary to combine the philosophy standing over the approaches mentioned above. Constructing a mathematical model we always have to simplify described process. This implies that we need to take a decision which parts of the described phenomena can be neglected and which not. Moreover, we need to choose a simplification level: either to describe a given process more precisely which would lead to a complex system of many equations with plenty of parameters that hardly can be analyzed by mathematical tools, or to simplify the description and reduce the number of variables/equations in order to give ourselves a chance for mathematical analysis and parameter identification. In the latter case it is sometimes useful to introduce time delays into the system in order to reflect that some processes take significantly more time than the others and to avoid more detailed descriptions.

As time delays can be observed even on the basic level of cellular interactions, we should also expect delays in interpersonal relationships. However, mathematical modeling in sociology and psychology is a rather young branch of applications, and dynamical systems (discrete or continuous) that have been used to describe some aspects of such relationships are typically simple models without delay, cf. e.g. [3,4,5,6,7,12,13,15,16,17,24] in the context of romantic relationships, and also the text-book by Gottman et al. [25] and the article written by Rey [11] focused on the prediction of marital dissolution on the basis of such sort of models. Typically, discrete dynamical systems approach is used in this topic, cf. [4,25]. Continuous dynamical systems in the description of romantic relationships has been introduced by Strogatz [16,26]. The idea of Strogatz influenced many researchers to study different styles of romantic relationships with linear and non-linear influence/ effort terms, cf. [5,7,11,12,13,15,17,24].

It is also worth to point that work by Strogatz also gave birth to a new branch of case study in which famous and well-described historical couples as well as characters from literature and pop-culture undergo analysis of relationship dynamics, cf. e.g. [12,14,27]. In frames of this approach one may find a broadened analysis of Petrarch's platonic feelings toward mistress Laura (true story which took place in 14th century), performed on the basis of facts extracted from historical records, and also characteristics of such fictional couples as Jack Dawson and Rose Calvert (*Titanic* movie), Dan Gallagher and Alexandra "Alex" Forrest (*Fatal Attraction* movie) or Christian de Neuvillette and Roxane (*Cyrano de Bergerac* by Rostand).

Recently, time delays have been introduced to such systems in order to reflect real interpersonal relations better, cf. [28,29,30] with the detailed discussion on introducing delays in [30]. The models presented and studied by Bielczyk et al. [29,30] essentially follow the ideas of Strogatz, because the assumption is for the interaction between partners to be based on linear interdependency with the time delay. On the other hand, linear modeling seems to be a far simplification for the complex problem of dynamics of interpersonal relationships, and nonlinear models appear to be more accurate here, cf. e.g. [5,8,9,10,11,12,13,14,17,25]. The nonlinear model with time delay in the influence terms was proposed and studied by Lion and Ran [28]. In this paper the authors considered the model with a general form of the functions reflecting the influence of the partner's love on the dynamics of each person's emotions and introduce time delays into it. They presented mathematical analysis of the model focusing on the local stability and Hopf bifurcation with respect to the sum of the delays. The analysis was performed under the assumption that, in the absence of any partner, the dynamics of each person is stable. Similar stability analysis can be found in [30], however in that paper all possible styles of romantic relationships were considered.

Among numerous ideas for developing Strogatz approach, there is another promising class of emerging models: some researchers pay attention to conscious activities of partners who moderate actions in a way to obtain an optimal outcome for themselves, from among all the given possibilities. In other words, partners in these models are described as intelligent agents capable of making decisions instead of being driven only by emotions. For example, [5] contributed to the economic theory of addictive behavior in a context of romantic relationships, by analysis of optimal control problem for a dyadic dynamical system. Another example can be found in [11], where it is shown that, whenever partners are similar in terms of emotional attributes, there is an optimal effort strategy leading to a stable and happy coexistence. This approach is interesting especially due to the idea of so called second law of thermodynamics for sentimental relationships which means a tendency for the initial feeling in the relationship to fade away with time if there is no prompt from agents, which must