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Efficient estimators of population mean using auxiliary attributes

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ABSTRACT

Abd-Elfattah et al. [1] suggested a set of estimators for calculating population mean using auxiliary attributes. This paper proposes a family of estimators based on an adaptation of the estimators presented by Koyuncu and Kadilar [2], and introduces a new family of exponential estimators using auxiliary attributes. The expressions of the mean square errors (MSEs) of the adapted and proposed families are derived in a general form. It is shown that the adapted version of the Koyuncu and Kadilar [2] estimators is always more efficient than that of Abd-Elfattah et al. [1]. Moreover, the new exponential estimators based on auxiliary attributes are more efficient than those of Koyuncu and Kadilar [2] and Abd-Elfattah et al. [1]. The theoretical findings are supported by a numerical example using original data.

1. Introduction and notation

In the sampling literature, auxiliary information is commonly used to improve estimates. Many authors have suggested estimators based on auxiliary information. One way to increase the efficiency of an estimator is to use auxiliary attributes. Naik and Gupta [3], Singh et al. [4], and Abd-Elfattah et al. [1] proposed a set of estimators using information from a single auxiliary attribute in a simple random sampling. Jhajj et al. [5] introduced a family of estimators, and Shabbir and Gupta [6] introduced a ratio-type estimator using an auxiliary attribute in a simple random sampling or a two-phase sampling. Shabbir and Gupta [7] suggested exponential-type estimators for use in two-phase sampling. In this paper, a family of estimators is developed by adapting the estimators of Koyuncu and Kadilar [2], and a new family of exponential estimators is introduced using the auxiliary attributes in a simple random sampling.

Let $U=\{U_1,U_2,\ldots,U_N\}$ be a finite population of size N. Let n be the size of the sample drawn from this population according to a simple random sampling without replacement. Let y_i and φ_i be the values of the study variable and auxiliary attribute, respectively, on the ith population unit ($i=1,2,\ldots,N$). Assume that the presence or absence of an attribute φ introduces a complete dichotomy into the population, and assume that the attribute φ takes only two values, 1 or 0. Accordingly,

$$\varphi_i = 1, \quad \text{if the ith unit of the population possesses attribute } \varphi,$$
$$\varphi_i = 0, \quad \text{otherwise.}$$

Let $A=\sum_{i=1}^{N}\varphi_i$ and $a=\sum_{i=1}^{n}\varphi_i$ denote, respectively, the total number of units in the population and in the sample possessing attribute φ. Let the corresponding population and sample proportions be $P=\frac{A}{N}$ and $p=\frac{a}{n}$, respectively. Let $\overline{Y}=\frac{1}{N}\sum_{i=1}^{N}y_i$ and $\bar{y}=\frac{1}{n}\sum_{i=1}^{n}y_i$ be the unknown population mean and sample mean of the study variable y, respectively. To obtain the MSE, let us define $\delta_y=(\bar{y}-\overline{Y})/\overline{Y}$ and $\delta_\varphi=(p-P)/P$. Using these notations, we have

$$E(\delta_y)=E(\delta_\varphi)=0,$$
$$E\left(\delta_y^2\right)=\frac{1-f}{n}C_y^2,\quad E\left(\delta_\varphi^2\right)=\frac{1-f}{n}C_p^2,\quad E(\delta_y\delta_\varphi)=\frac{1-f}{n}C_{y\varphi}=\frac{1-f}{n}\rho_{\text{Pb}}C_yC_\varphi,$$

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where

$$C_y^2=\frac{S_y^2}{\overline{Y}^2},\quad C_p^2=\frac{S_\varphi^2}{P^2},\quad C_{y\varphi}=\frac{S_{y\varphi}}{\overline{Y}P},$$
$$S_\varphi^2=\frac{1}{N-1}\sum_{i=1}^{N}(\varphi_i-P)^2,\quad S_{y\varphi}=\frac{1}{N-1}\sum_{i=1}^{N}(y_i-\overline{Y})(\varphi_i-P).$$

The Naik and Gupta [3] ratio estimator is given by

$$t_{\mathrm{NG}}=\bar{y}\frac{P}{p}. \tag{1}$$

The MSE of t_{NG}, to the first order of approximation, is given by

$$\mathrm{MSE}(t_{\mathrm{NG}})=\frac{1-f}{n}\left(S_y^2+R_1^2S_\varphi^2-2R_1S_{y\varphi}\right), \tag{2}$$

where $R_1=\frac{\overline{Y}}{P}$.

Singh et al. [4] introduced a set of estimators using the known parameters of an auxiliary attribute. These estimators are given by

$$t_1=\frac{\bar{y}+b_\varphi(P-p)}{p}P, \tag{3}$$
$$t_2=\frac{\bar{y}+b_\varphi(P-p)}{p+B_2(\varphi)}(P+B_2(\varphi)), \tag{4}$$
$$t_3=\frac{\bar{y}+b_\varphi(P-p)}{p+C_p}(P+C_p), \tag{5}$$
$$t_4=\frac{\bar{y}+b_\varphi(P-p)}{pB_2(\varphi)+C_p}[PB_2(\varphi)+C_p], \tag{6}$$
$$t_5=\frac{\bar{y}+b_\varphi(P-p)}{pC_p+B_2(\varphi)}[PC_p+B_2(\varphi)], \tag{7}$$

where C_p, $B_2(\varphi)$, and b_φ are, respectively, the population coefficient of variation, population coefficient of kurtosis of the auxiliary attribute, and the regression coefficient.

The MSE of t_i s (i = 1,2,...,5), to the first order of approximation, is given by

$$\mathrm{MSE}(t_i)=\frac{1-f}{n}\left[R_i^2S_\varphi^2+S_y^2(1-\rho_{\mathrm{Pb}}^2)\right], \tag{8}$$

where

$$R_1=\frac{\overline{Y}}{P},\quad R_2=\frac{\overline{Y}}{P+B_2(\varphi)},\quad R_3=\frac{\overline{Y}}{P+C_p},\quad R_4=\frac{\overline{Y}B_2(\varphi)}{PB_2(\varphi)+C_p},\quad R_5=\frac{\overline{Y}C_p}{PC_p+B_2(\varphi)}.$$

Abd-Elfattah et al. [1] suggested estimators that combined those of Singh et al. [4]. These estimators are given by

$$t_{\mathrm{pro1}}=m_1\frac{\bar{y}+b_\varphi(P-p)}{p}P+m_2\frac{\bar{y}+b_\varphi(P-p)}{p+B_2(\varphi)}(P+B_2(\varphi)), \tag{9}$$
$$t_{\mathrm{pro2}}=m_1\frac{\bar{y}+b_\varphi(P-p)}{p}P+m_2\frac{\bar{y}+b_\varphi(P-p)}{p+C_p}(P+C_p), \tag{10}$$
$$t_{\mathrm{pro3}}=m_1\frac{\bar{y}+b_\varphi(P-p)}{p}P+m_2\frac{\bar{y}+b_\varphi(P-p)}{pB_2(\varphi)+C_p}(PB_2(\varphi)+C_p), \tag{11}$$
$$t_{\mathrm{pro4}}=m_1\frac{\bar{y}+b_\varphi(P-p)}{p}P+m_2\frac{\bar{y}+b_\varphi(P-p)}{pC_p+B_2(\varphi)}(PC_p+B_2(\varphi)), \tag{12}$$

where m_1, m_2 are weights that satisfy the condition $m_1+m_2=1$. The MSE associated with the t_{proi} values (i = 1,2,...,4), to the first order of approximation, is given by

$$\mathrm{MSE}(t_{\mathrm{proi}})=\left(\frac{1-f}{n}\right)\left(S_y^2-2\eta S_{y\varphi}+\eta^2S_\varphi^2\right), \tag{13}$$

where $\eta=m_1(R_1+B_\varphi)+m_2(R_2+B_\varphi)$. The optimal values of m_1 and m_2, are given by

$$m_1^*=\frac{R_2}{R_2-R_1},\quad m_2^*=\frac{R_1}{R_1-R_2}, \tag{14}$$