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Asymptotic behavior for an Ohmic heating model in thermal electricity ☆

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ABSTRACT

In this paper, we consider the asymptotic behavior for a nonlocal parabolic problem with Dirichlet boundary condition, which is raised from the thermal-electricity and is so-called an Ohmic heating model. It models the system of the temperature of a conductor in the device, which is connected in series with another conductor with constant. The electrical resistivity of the one of the conductors depends on the temperature and the other one remains constant. An analysis of the nonlinear problem shows that the solution exists global and the unique stationary solution is globally asymptotically stable. The results assert that the temperature of the conductor remains bounded and the system converges asymptotically to the unique equilibrium.

1. Introduction

In this paper, we study the asymptotic behavior for the following one dimensional nonlocal parabolic problem

$$\begin{cases} u_t = u_{xx} + \dfrac{\rho(u)}{\left(a+b\int_{-1}^{1}\rho(u)dx\right)^2}, & (x,t)\in(-1,1)\times(0,\infty),\\ u(\pm1,t)=0, & t>0,\\ u(x,0)=u_0(x), & x\in(-1,1), \end{cases} \tag{1.1}$$

where a, b are positive parameters, defined as (4.9) in the Appendix, and $u_0(x)$ is a positive bounded continuous function.

The original form of the problem (1.1) arises in an Ohmic heating model, which comes from the thermal electricity (see [1–3]), written as

$$\left\{ u_t = \Delta u + \sigma(u)|\nabla\varphi|^2, \quad \operatorname{div}(\sigma(u)\nabla\varphi)=0. \right.$$

The variable u describes the dimensionless temperature of the conductor A. Suppose the electrical resistivity of the conductor B remains almost a constant $R_0>0$, while the one of the conductor A varies significantly with its temperature u, written as $\rho=\rho(u)$ (see Fig. 1). $\varphi=\varphi(x,y,z,t)$ is the electric potential and $\sigma(u)=1/\rho(u)$ represents the electrical conductivity. For the convenience to the readers, we will give the derivation of the model (1.1) in the Appendix.

In general, for the nature of the conducting medium, the resistivity is either a decreasing or increasing function of the temperature, such as fuse wire, electric arcs, thermistor etc. In this paper, we assume that the function ρ is continuous, positive and decreasing, namely,

$$\rho(s)>0, \rho'(s)<0, \quad \text{for all } s\geqslant 0, \tag{1.2}$$

which permit us to use the comparison argument in the analysis of the model mathematically.

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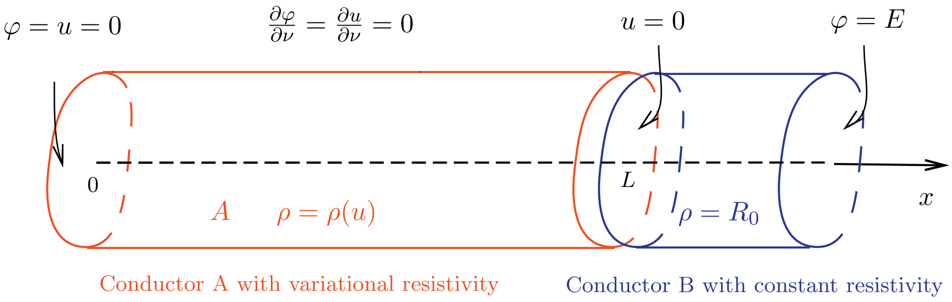

Fig. 1. Electric current flows through two conductors.

In [10,11], Lacey considered the following model for only one conductor A,

$$u_t = \Delta u + \frac{\lambda\rho(u)}{\left(\int_\Omega \rho(u)dx\right)^2}, \quad x \in \Omega,\ t > 0, \tag{1.3}$$

where Ω is an open, bounded domain in $\mathbb{R}^2$. Furthermore, they proved the occurrence of blow-up for the one-dimensional problem with Dirichlet boundary conditions. At the end of his work ([10]), Lacey mentioned the model (1.1) and claimed that the solution of the problem (1.1) exists globally in time. Inspired by these works, the main purpose of this paper is to give the asymptotic behavior and to show the asymptotical stability of global solution of the problem (1.1).

Our main results read as follows.

Theorem 1.1. *Assume the conditions (1.2) hold, the solution of the problem (1.1) converges asymptotically to the steady state, namely,*

$$u(x,t) \to \omega(x), \quad \textit{as } t \to +\infty, \quad \text{for } x \in (-1,1),$$

where $\omega(x)$ is the unique solution of the corresponding steady problem of (1.1) for any decreasing $\rho(u)$.

Remark 1.1. For an example, $\rho(u) = e^{-u}$, by a direct computation, the explicit form of the unique equilibrium state of the problem (1.1) is

$$\omega(x) = 2\ln\frac{\cos\alpha x}{\cos\alpha}, \quad \text{for } x \in (-1,1),$$

where $\alpha \in (0, \pi/2)$ is the unique root of the equation

$$\sqrt{2}a\alpha + 2\sqrt{2}b\sin\alpha\cos\alpha - \cos\alpha = 0.$$

Remark 1.2. There are many literatures for considering the nonlocal parabolic equations, the authors would like to refer Refs. [5–8,13,15,16] to the readers.

Remark 1.3. In [11], Lacey gave the asymptotic behavior of the global solution and some blow-up results to one conductor model (1.3) for special case $\rho(u) = e^{-u}$. However, in present model (1.1), the conductor A in series with a constant conductor B remains always bounded. Furthermore, we prove the temperature of the asymptotic behavior of the global solution in more general conductivity-temperature relationship $\rho(u)$.

The remain of the paper is organized as follows. In Section 2, we will give some preliminary results such as the local existence, global existence, comparison principle of classical solution to the problem (1.1). In Section 3, we will prove the asymptotic behavior of the solution to the problem (1.1). In the Appendix, we give the details for the derivation of the model (1.1).

2. Preliminary results

In this section, we give some preliminary results to the Dirichlet problem (1.1). Firstly, we consider the local existence, uniqueness and monotonicity of the solution to (1.1).

Proposition 2.1. *For any positive constants a, b and nonnegative bounded initial data $u_0(x)$, suppose that the function $\rho(u)$ is continuous and positive, then the problem (1.1) has a unique classical solution in $Q_T = [-1,1] \times [0,T)$ for some $T > 0$.*