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Torsional surface wave propagation in an initially stressed non-homogeneous layer over a non-homogeneous half-space

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ABSTRACT

It is of great interest to study torsional surface wave propagation in an initially stressed non-homogeneous layer over a non-homogeneous half-space. The method of separation of variables is applied to find the displacement field. It is well known in the literature that the earth medium is not at all initial stress free and homogeneous throughout, but it is initially stressed and non-homogeneous. Keeping these things in mind, we have discussed propagation of torsional surface wave in an initially stressed non-homogeneous layer over a non-homogeneous half-space. It has been observed that the inhomogeneity parameter and the initial stress play an important role for the propagation of torsional surface wave. It has been seen that as the non-homogeneity parameter in the layer increases, the velocity of torsional surface wave also increases. Similarly as the non-homogeneity parameter in the half-space increases, the velocity of torsional surface wave increases. The initial stresses P present in the inhomogeneous layer also have effect in the velocity of propagation. It has been observed that an increase in compressive initial stresses decreases the velocity of torsional surface wave.

1. Introduction

The study of seismic waves gives important information about the layered earth structure and has been used to accurately determine the earthquake epicenter. Earthquakes generate waves on the grandest scale, with surface waves observable after several trips around the world, and their systematic study has obvious implications for man's safety, as well as for his curiosity concerning the structure and evolution of the earth. Artificially generated seismic waves provide information about the configuration of rock layers for oil exploration and, on a smaller scale, information as to the rigidity of shallow layers for engineering purposes. Properties of rocks penetrated by oil wells have been determined by observing seismic waves at various depths, due either to a distant explosion or to a sound source nearby in the same well. Hence, the study of the surface waves and their propagation in various media is of great geophysical significance. Thus, modeling of seismic wave propagation plays a significantly important role and is of great utility in the exploration of petroleum, earthquake disaster prevention, civil engineering and signal processing. Ewing et al. [1] has given the basic literature on the propagation of elastic waves. A large number of papers have been published in different journals after the publication of this book. A detail study on elastic wave propagation and its generation in seismology had been made by Pujol [2] and Chapman [3]. Recently a lot of work has been done by many researchers in the field of wave propagation. Some of them are Singh [4], Ponnusamy and Rajagopal [5], Nayfesh and Abdelrahman [6], Tomar and Singh [7], Asfar and Hawwa [8] and Tomar [9].

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The problems related to initial stressed elastic medium has been a subject of continued interest due to its importance in various fields, such as earthquake engineering, seismology and Geophysics. The development of initial stresses in the medium is due to many reasons, for example, resulting from the difference of temperature, process of quenching, shot peening and cold working, slow process of creep, differential external forces, gravity variation etc. It is therefore of much interest to study the influence of these stresses on the propagation of surface waves. These stresses have a pronounced influence on the propagation of waves as shown by Biot [10]. Biot showed that the acoustic wave propagation under initial stress was fundamentally different from the stress free case and could not be represented by simply introducing into the classical theory and the stress dependent elastic coefficients. In his treatment he considered the fluid as a particular case of an elastic medium under initial stress with rigidity zero. A detailed discussion based on this viewpoint is found in some of his remarkable papers Biot [11–13]. Several authors have employed the theory of incremental deformation formulated by Biot [14] to study the propagation of surface waves in pre-stressed elastic solids. Cauchy [15] used assumption that stress was due to central forces between particles of the solid. Bromwich [16] examined the efforts of gravity on surface waves. The case of uniform initial stress has been discussed by Southwell [17]. The equations for an incompressible solid under hydrostatic pressure have been derived by Love [18].

A study of the effect of inhomogeneity on the propagation of surface waves provides an interesting field for the application of mathematical technique and in addition, is of practical importance to seismologists because in any realistic model of the earth there is a continuous change in the elastic properties of the material in the vertical direction giving rise to inhomogeneity.

Torsional surface wave is one kind of surface wave which is horizontally polarized but gives a twist to the medium when it propagates. It has been confirmed that although homogeneous elastic half-space does not allow torsional surface waves to propagate but certain types of non-homogeneity in the layers allows it to propagate. Meissner [19] has shown that an inhomogeneous elastic half-space with a quadratic variation of shear modulus and density varying linearly with depth, the torsional surface waves do exist. It was pointed out by Rayleigh [20] that an isotropic homogeneous elastic half-space doesn't allow torsional surface waves to propagate. Vardoulakis [21] has studied that torsional surface waves also propagate in Gibson's half-space where the shear modulus varies linearly with depth but the density remains constant. Georgiadis et al. [22] have demonstrated that torsional surface waves do exist in a gradient elastic half-space. Selim [23] has discussed the propagation of torsional surface waves in heterogeneous half-space with irregular free surface. The propagation of torsional surface waves in an elastic half-space with void pores has been studied by Dey et al. [24]. The propagation of torsional surface wave in an initially stressed cylinder has been discussed by Dey and Dutta [25]. The torsional wave propagation in a two-layered circular cylinder with imperfect bond has been investigated by Bhattacharya [26]. Paul and Sarma [27] have discussed the propagation of torsional wave in a finite piezoelectric cylindrical shell. Propagation of torsional surface waves in a homogeneous layer of finite thickness over an initially stressed heterogeneous half-space has been studied by Gupta et al. [28]. Barton et al. [29] have studied nonlinear dynamics of torsional waves in a drill-string model with spatial extent. The earth is considered to be a layered elastic medium with variation in density and rigidity in constituent layers. Therefore the torsional surface wave must propagate during earthquakes. The near surface of the earth consists of layers of different types of material properties overlying a half-space of various types of rock, underground water, oil and gases. So, the studies of the propagation of torsional surface waves will be of great interest to seismologists.

In the present paper torsional wave propagation in anisotropic initially stressed layer of sandstone over a non-homogeneous half-space has been studied. The inhomogeneity of the layer is taken into consideration by assuming $N = N_0 \cosh^2 \alpha z$, $L = L_0 \cosh^2 \alpha z$, $\rho = \rho_0 \cosh^2 \alpha z$ *and* $P = P_0 \cosh^2 \alpha z$ where N, L are directional rigidities, ρ is the density, P is the compressive initial stress at any point in the layer which is assumed to be transversely isotropic with z-axis as the axis of symmetry and α is a constant having dimension that is inverse of length. The inhomogeneity of the half-space has been taken along the z-direction. In the half-space polynomial variation in rigidity and density with depth has been considered. The velocities of torsional waves are obtained as complex ones, in which real part gives the phase velocity of propagation and corresponding imaginary part gives the damping.

2. Formulation

For the study of torsional surface waves, a cylindrical co-ordinate system has been considered. The model consists of a non-homogeneous anisotropic layer of finite thickness H under compressive initial stress P along the radial direction and over an inhomogeneous elastic half-space. The interface is located at $z = 0$ and the z-axis is directed vertically downward as shown in Fig. 1. N, L are the directional rigidities, ρ is the density and P is the compressive initial stress at any point in the layer which is assumed to be transversely isotropic with z-axis as the axis of symmetry.

Consider the hyperbolic variation in elastic moduli, density and initial stress with depth z as

$$\left.\begin{aligned} N &= N_0 \cosh^2 \alpha z \\ L &= L_0 \cosh^2 \alpha z \\ \rho &= \rho_0 \cosh^2 \alpha z \\ P &= P_0 \cosh^2 \alpha z \end{aligned}\right\} \tag{1}$$