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A parallel recurrence method for the fast computation of Zernike moments

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ABSTRACT

This paper presents a parallel recursive method for the computation of Zernike moments from a digital image. The proposed method can reduce the computational complexity of the Zernike radial polynomials by introducing a novel recurrence relation, and be applicable to either the computation of a single Zernike moment or entire set of Zernike moments. The fast computation is achieved because it involves less addition and multiplication operations and is executed in parallel. Moreover, the single Zernike moment can be obtained with employing partial Zernike moments of lower orders. The experiments are carried out to evaluate the performance of the proposed method using binary and grayscale images. The experimental results show that the proposed method takes the shortest time in computing the Zernike moments of a specific order $\leqslant 28$ as well as the entire Zernike moments of orders $\leqslant 70$.

1. Introduction

First introduced in image analysis by Teague [1], Zernike moments have been widely applied in pattern recognition [2–6], image and texture retrieval [7–11], watermarking [12–16], texture and image analysis [17–20], biometrics [21–24], biomedical engineering [25–27] and edge detection [28]. Zernike moments have be proved to be better than the other types of moments such as Legendre, Geometric and rotational moments in terms of the feature representation capabilities and robustness in the presence of a moderate level of noise [1,29,30]. As these Zernike polynomials are orthogonal to each other, it can help in achieving a near no redundancy or overlap of information between the moments [1,29]. In addition, the Zernike orthogonal moments are invariant under rotation and the simplicity of image reconstruction [1,30]. Due to these characteristics, the scientific interest in engineering is still increased.

Unfortunately, the straightforward method takes excessive amount of time for computation of Zernike moments. To overcome this shortcoming, many fast methods for Zernike moments calculation have been developed at the cost of loss of accuracy. Mukundan and Ramakrishnan [31] has proposed fast algorithms for computation of Zernike moments based on square-to-circular transformation. The transformation helps in a much faster integration of intensity values along contours of equal radii, but the limitations is that it is only suitable for binary image. Moreover, the quality of reconstructed images is degraded due to the square-to-circular transformation method. As the factorial functions in the radial polynomials contribute significantly towards computation of Zernike moments, some researchers have tried to avoid computing the factorials by introducing a recurrence relation. Prata and Rusch [9] and Kintner [32] have proposed a recurrence relation for radial polynomials of Zernike moments respectively. However, some values for these methods must be directly computed

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from the definition of Zernike radial polynomials, which increases excessive computation cost. In order to further improve speed of computation, Chong et al. [33] has proposed a modified Kintner's method. The technique is applicable in these cases where ($p = q$) and ($p = q + 2$). Chong also proposed a q-recursive method based on recurrence relation of radial polynomials. Among all recurrence methods, the q-recursive method shows the fastest performance to compute a single order because it enables the Zernike moments of a specific order to be obtained without employing lower or higher orders. Some methods [34,35] also utilize the symmetric or anti-symmetric properties of Zernike moments kernel functions to enhance the speed of computation. A significant improved method that combines the symmetry and anti symmetry properties is proposed by Hwang and Kim [35]. This method can be combined with other existing fast methods such as q-recursive method and modified Prata's method, and these mixed methods show better performance. Recently, Hosny [36] proposes a systematic method for fast and accurate computation of full and subsets Zernike moment. The method displays better advantages in accuracy and speed. Singh and Walia [37] reinvestigates the Prata's recursive method and proposes a method called the modified Prata's method. A set of orders of Zernike moments is fast computed as the method involves the least number of operations in each recursion. In addition, to reduce geometric error and numerical integration error, they also proposed an approach for computation of high order Zernike moments [38].

In this paper, we proposed a novel approach for fast computation of Zernike moments. The aim of the algorithm is to reduce the computation time further by the recursive property of Zernike moments. The proposed method uses Zernike radial polynomials with higher index q and lower index p to derive the polynomial of the lower index q and higher index p. In addition, the entire set of any p with varying q of Zernike moments can be obtained with only using partial lower orders. It is useful to compute a single order of orders of Zernike moments. The number of arithmetic operations involved in the proposed method is approximately the same as that involved in modified Prata's method. Moreover, the proposed recurrence method is easy to be executed in parallel. Hence, the time taken to compute a single order and a set of orders of Zernike is reduced effectively. The previous and proposed methods are compared in our numerical experiments. The results show that the proposed method takes the shortest time to compute a specific order $\leqslant 28$ and exhibits very fast speed for computation of the entire Zernike moments of orders $\leqslant 70$.

The outline of this paper is as follows: The next section concisely presents Zernike moments. The current recursive fast methods for Zernike moments calculation are reviewed in the Section 3. In Section 4, a fast method for the computation of Zernike moments is detailed. Section 5 analyzes the time complexity of several existing methods. Section 6 shows the experimental results. The last section concludes the paper.

2. The Zernike moments

The kernel function of Zernike moments comprises orthogonal Zernike polynomials defined over the polar coordinate space inside a unit circle. Zernike moments are the projection of the image function onto these orthogonal basis functions. The two-dimensional Zernike moments of order p with repetition q for a continuous image function $f(x,y)$ that vanishes outside the unit circle are defined by

$$A_{pq} = \frac{p+1}{\pi}\int_0^{2\pi}\int_0^1 f(\rho,\theta)V_{pq}^*(\rho,\theta)\rho d\rho d\theta \tag{1}$$

Using the radial polynomial $R_{pq}(\rho)$, the basis functions of Zernike moments are formed by

$$V_{pq}(\rho,\theta) = R_{pq}(\rho)e^{\bar{j}q\theta} \tag{2}$$

Where, $\bar{j} = \sqrt{-1}$, $p \geqslant 0$, $|q| \leqslant p$, $p - |q| = even$.

Radial polynomial $R_{pq}(\rho)$ defined as

$$R_{pq}(\rho) = \sum_{s=0}^{(p-|q|)/2}(-1)^s \bullet \frac{(p-s)!}{s!\left(\frac{p+|q|}{2}-s\right)!\left(\frac{p-|q|}{2}-s\right)!}\rho^{p-2s} \tag{3}$$

Note that $R_{p,-q}(\rho) = R_{pq}(\rho)$

These polynomials are orthogonal and satisfy

$$\int_0^{2\pi}\int_0^1 V_{pq}^*(\rho,\theta)V_{nm}(\rho,\theta)\rho d\rho d\theta = \begin{cases}\frac{\pi}{n+1} & \text{if} \quad p=n,\ q=m \\ 0 & \text{otherwise}\end{cases} \tag{4}$$

The orthogonality implies the minimum a mount of information redundancy for representation of image, and enables the individual contribution of each order moment to be separable in the reconstruction process. Hence, the reconstruction image can be obtained by adding the individual components of each order.

The reconstruction process is expressed as

$$f(\rho,\theta) = \sum_{p=0}^{\infty}\sum_{q=-p}^{p}A_{pq}V_{pq}(\rho,\theta) \tag{5}$$