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Classification of single traveling wave solutions to the nonlinear dispersion Drinfel'd–Sokolov system

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ABSTRACT

By the complete discrimination system for polynomial method, we obtained the classification of single traveling wave solutions to the nonlinear dispersion Drinfel'd–Sokolov system.

1. Introduction

Many nonlinear differential equations which can be directly reduced to the integral forms under the traveling wave transformation, can be solved by some approaches such as the transformed rational function method [1], the multiple exp-function method [2,3], Frobenius decomposition method [4], the complete discrimination system for polynomial method [5–11], and so on.

In this paper, we consider the following nonlinear dispersion Drinfel'd–Sokolov system (D (m,n)):

$$\begin{cases} u_t + (v^m)_x = 0, \\ v_t + a(v^n)_{xxx} + bu_x v + cuv_x = 0, \end{cases} \tag{1}$$

where a, b, c, m, n are real constant parameters.

More recently, Eq. (1) has been studied by some authors [12–16]. In Ref. [14], some solutions of Eq. (1) were given by bifurcation method of dynamical systems. In Ref. [15], many types of compaction and solitary pattern solutions of Eq. (1) were obtained by using some transformations. In Ref. [16], more new exact traveling wave solutions of the Eq. (1) were obtained by using the Weierstrass elliptic function method.

In this paper, by using Liu's complete discrimination system for polynomial method [11], we obtained the classification of single traveling wave solutions to Eq. (1), for $n = 1,\ m = 1, 2, 3$ and $n = 2,\ m = 1, 2$. All the solutions have been verified that they are really solutions of Eq. (1) by Mathematica.

2. The integral forms of Eq. (1)

First, under the traveling wave transformation $u(x,t) = u(\xi),\ v(x,t) = v(\xi),\ \xi = x - \lambda t$, we have

$$\begin{cases} -\lambda u' + (v^m)' = 0, \\ -\lambda v' + a(v^n)''' + bu'v + cuv' = 0. \end{cases} \tag{2}$$

Second, integrating the first equation of Eq. (2) once, we have

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$$u = c_0 + \frac{v^m}{\lambda} \tag{3}$$

and substituting it into the second equation of Eq. (2), yields the following ordinary differential equation

$$v'' + \frac{n-1}{v}(v')^2 + \frac{bm+c}{a\lambda(m+1)n} v^{m-n+2} + \frac{c_0 c - \lambda}{an} v^{2-n} + c_1 v^{1-n} = 0. \tag{4}$$

Third, integrating Eq. (4) once, we can get the elementary integral form of Eq. (4):

$$\begin{aligned} &\pm(\xi - \xi_0) = \int \frac{dv}{\sqrt{f(v)}}, \\ &f(v) = \frac{c_2 - \frac{2c_1}{n} v^n - \frac{2(c+bm)}{a\lambda n(m+1)(m+n+1)} v^{m+n+1} - \frac{2(cc_0-\lambda)}{an(n+1)} v^{n+1}}{v^{2n-2}}, \end{aligned} \tag{5}$$

where $c_0,\ c_1,\ c_2$ are integral constants.

By the above analysis, for given parameters, calculating the Eq. (5), we can obtain v. Then substituting v into Eq. (3), we can obtain u, thus the solution of Eq. (1) will be obtained.

3. Classification of the solution v with $n = 1$

When $n = 1$, Eq. (5) reads

$$\pm(\xi - \xi_0) = \int \frac{dv}{\sqrt{f(v)}}, \quad f(v) = a_0 + a_1 v + a_2 v^2 + a_{m+2} v^{m+2}, \tag{6}$$

where $a_0 = c_2,\ a_1 = -2c_1,\ a_2 = \frac{\lambda - cc_0}{a},\ a_{m+2} = -\frac{2(bm+c)}{a\lambda(m+1)(m+2)}$.

Case 3.1: $m = 1,\ b = -c$. According to Eq. (6), we have $f(v) = a_0 + a_1 v + a_2 v^2$. The discrimination of $f(v)$ is $\Delta = a_1^2 - 4a_0 a_2$.

Case 3.1.1: $\Delta = 0$. If $a_2 > 0$, then

$$v = \pm \exp\left(\pm\sqrt{a_2}(\xi - \xi_0)\right) - \frac{a_1}{2a_2}. \tag{7}$$

Case 3.1.2: $\Delta > 0$. If $a_2 > 0$, then

$$v = \pm\frac{\sqrt{\Delta}}{2a_2} \sinh\left(\sqrt{a_2}(\xi - \xi_0)\right) - \frac{a_1}{2a_2}. \tag{8}$$

If $a_2 < 0$, then

$$v = \pm\frac{\sqrt{\Delta}}{2a_2} \sin\left(\sqrt{-a_2}(\xi - \xi_0)\right) - \frac{a_1}{2a_2}. \tag{9}$$

Case 3.1.3: $\Delta < 0$. If $a_2 > 0$, then

$$v = \pm\frac{\sqrt{-\Delta}}{2a_2} \sinh\left(\sqrt{a_2}(\xi - \xi_0)\right) - \frac{a_1}{2a_2}. \tag{10}$$

Case 3.2: $m = 1,\ b \neq -c$. According to Eq. (6), we have $f(v) = a_0 + a_1 v + a_2 v^2 + a_3 v^3$. Taking the transformation

$$w = a_3^{\frac{1}{3}}\left(v + \frac{a_2}{3a_3}\right), \quad \xi_1 = a_3^{\frac{1}{3}}\xi, \tag{11}$$

we have

$$\pm(\xi_1 - \xi_0) = \int \frac{dw}{\sqrt{f(w)}}, \quad f(w) = w^3 + b_1 w + b_0, \tag{12}$$

where $b_0 = a_0 + \frac{2a_2^3 - 9a_1 a_2 a_3}{27a_3^2},\ b_1 = \frac{a_2^2 + 3a_1 a_3}{3a_3^{\frac{4}{3}}}$.

Denote

$$\Delta = -\left(\frac{b_0^2}{4} + \frac{b_1^3}{27}\right). \tag{13}$$

Δ and b_1 make up the complete discrimination system of $f(w)$.

Case 3.2.1: $\Delta = 0,\ b_1 < 0$, we have $f(w) = (w-\alpha)^2(w-\beta)$, where $\alpha,\ \beta$ are real and $\alpha \neq \beta$. If $w > \alpha > \beta$, then