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Multiobjective nonlinear second order symmetric duality with (K,F)-pseudoconvexity

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ABSTRACT

The purpose of this paper is to introduce a new class of generalized cone-pseudoconvex functions and strongly cone-pseudoconvex functions, called second order (K,F)-pseudoconvex functions and strongly second order (K,F)-pseudoconvex functions. A pair of second order symmetric dual multiobjective nonlinear programs is formulated over arbitrary generalized cone-pseudoconvex functions. For these second order symmetric dual programs, the weak, strong and converse duality theorems are established using the above generalization of cone-pseudoconvex functions. A self duality theorem is also given by assuming the functions involved to be skew-symmetric.

1. Introduction

Duality is an important concept in the study of nonlinear programming. Symmetric duality in nonlinear programming in which the dual of the dual is the primal was first introduced by Dorn [4]. Subsequently Dantzig et al. [3] established symmetric duality results for convex/concave functions with nonnegative orthant as the cone. The symmetric duality result was generalized by Bazaraa and Goode [1] to arbitrary cones. Kim et al. [9] formulated a pair of multiobjective symmetric dual programs for pseudoinvex functions and arbitrary cones. The weak, strong, converse and self duality theorems were established for that pair of dual models.

The study of second order duality is significant due to the computational advantage over first order duality as it provides tighter bounds for the value of the objective function when approximations are used (see [5,11,12]).

Hou and Yang [5] introduced a pair of second order symmetric dual non-differentiable programs and second order F-pseudoconvex and proved the weak and strong duality theorems for these second order symmetric dual programs under the F-pseudoconvex assumption. Suneja et al. [10] formulated a pair of multiobjective symmetric dual programs over arbitrary cones for cone-convex functions. The weak, strong, converse and self-duality theorems were proved for these programs. Yang et al. [11] formulated a pair of Wolf type non-differentiable second order symmetric primal and dual problems in mathematical programming. The weak and strong duality theorems were established under second order F-convexity assumptions. Symmetric minimax mixed integer primal and dual problems were also investigated. Khurana [8] introduced cone-pseudoinvex and strongly cone-pseudoinvex functions, and formulated a pair of Mond–Weir type symmetric dual multiobjective programs over arbitrary cones. The duality theorems and the self-dual theorem were established under these functions. Yang et al. [12] proved the weak, strong and converse duality theorems under F-convexity conditions for a pair of second order symmetric dual programs. Yang et al. [13] established various duality results for nonlinear programming with cone constraints and its four dual models introduced by Chandra and Abha [2].

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In this paper, we present new definitions for generalized cone-pseudoconvex functions, called second order (K,F)-pseudoconvex functions and strongly second order (K,F)-pseudoconvex functions. We suggest a pair of multiobjective nonlinear second order symmetric dual programs and establish the duality theorems using the above generalization of cone-pseudoconvex functions. Finally, a self-duality theorem is given by assuming the skew-symmetric of the functions.

2. Notations and definitions

The following conventions for vectors in R^n will be used:

$$x < y \Longleftrightarrow x_i < y_i, \quad i = 1,2,\ldots,n,$$
$$x \leqq y \Longleftrightarrow x_i \leqq y_i, \quad i = 1,2,\ldots,n,$$
$$x \leqslant y \Longleftrightarrow x_i \leqq y_i, \quad i = 1,2,\ldots,n \text{ but } x \neq y.$$

A general multiobjective nonlinear programming problem can be expressed in the form:

$$\min f(x) = (f_1(x), f_2(x), \ldots, f_m(x)) \tag{P}$$
$$\text{subject to } x \in X = \{x \in R^n | g_j(x) \leqq 0, \quad j = 1,2,\ldots,k\},$$

where

$$f : R^n \to R^m \text{ and } g : R^n \to R^k.$$

Definition 1. A point $\bar{x} \in X$ is said to be an efficient (or a Pareto optimal) solution of problem (P) if there exists no other $x \in X$ such that $f(x) \leqslant f(\bar{x}), \quad (f_i(x) \leqq f_i(\bar{x}), \quad i = 1,2,\ldots,m$ but $f(x) \neq f(\bar{x}))$.

To give the desired definitions, second order (K,F)-pseudoconvex and strongly second order (K,F)-pseudoconvex functions, let us first recall the following definitions which are due to the references [5–7,9,10]:

Definition 2 ([11,12]). A functional F: $X \times X \times R^n \to R(X \subseteq R^n)$ is sublinear in its third component if, for all $(x,u) \in X \times X$,

(i) $F(x,u;a_1 + a_2) \leqq F(x,u;a_1) + F(x,u;a_2) \forall a_1, a_2 \in R^n$; and
(ii) $F(x,u;\alpha a) = \alpha F(x,u;a) \ \forall a \in R^n, \forall \alpha \in R, \alpha \geq 0$.

For notational convenience, we write

$$F_{x,u}(a) = F(x,u;a).$$

Let K be a closed convex pointed cone in R^m with $int\, K \neq \phi$ and f:$R^n \to R^m$ be a differentiable function.

Definition 3 ([6,8]). The polar cone K^* of K is defined as

$$K^* = \{z \in R^m | x^T z \geqq 0 \ \forall x \in K\}.$$

Definition 4 [5]. The function f is said to be second-order F-pseudoconvex at $u \in X$ if $(x,p) \in X \times R^n$,

$$F_{x,u}[\nabla_u f(u) + \nabla_{uu} f(u)p] \geqq 0 \Rightarrow f(x) \geqq f(u) - \frac{1}{2} p^T \nabla_{uu} f(u)p.$$

f is second-order F-pseudoconcave if $-f$ is second-order F-pseudoconvex.

Now, we are in position to give our definitions of second-order (K,F)-pseudoconvex functions and strongly second-order (K,F)-pseudoconvex functions.

Definition 5. The function f is said to be second-order (K,F)-pseudoconvex at $u \in X$ if $(x,p) \in X \times R^n$,

$$-F_{x,u}[\nabla_u f(u) + \nabla_{uu} f(u)p] \notin \text{int } K \Rightarrow -\left[f(x) - f(u) + \frac{1}{2} p^T \nabla_{uu} f(u)p\right] \notin \text{int } K;$$

and the function f is said to be strongly second-order (K,F)-pseudoconvex at $u \in X$ if $(x,p) \in X \times R^n$,

$$-F_{x,u}[\nabla_u f(u) + \nabla_{uu} f(u)p] \notin \text{int } K \Rightarrow f(x) - f(u) + \frac{1}{2} p^T \nabla_{uu} f(u)p \in K.$$

f is second-order (K,F)-pseudoconcave if $-f$ is second-order (K,F)-pseudoconvex and f is strongly second-order (K,F)-pseudoconcave if $-f$ is strongly second-order (K,F)-pseudoconvex.